

MODELLING OF WEATHER
DATA TO BE USED IN THE
CMHC-2 PROGRAM

PREPARED FOR
THE TECHNICAL RESEARCH DIVISION
POLICY DEVELOPMENT AND RESEARCH SECTOR
OF
CANADA MORTGAGE AND HOUSING CORPORATION

BY
L. JOHN PARSONS
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CMHC PROJECT MANAGER
W. WEBSTER

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SUMMARY

The long-term hourly drybulb temperature data for Ottawa, Vancouver and Saskatoon were mathematically analyzed, in order to develop simple correlations which can be used in the CMHC-2 program to model the thermal performance of houses.

The relationships developed, will significantly improve the precision of thermal heat loss analysis in the program.

The best approach, (the power curve), was chosen based on a rigorous mathematical analysis and computer simulation of the alternative mathematical techniques.

The recommended technique is simple to implement, produces accurate results, is concise in its storage requirements and is comprehensive in its ability to define the temperature differential causing heat flow through a building envelope.

Further work is required to develop mathematical relationships to define the heating temperature differentials for 58 other Canadian sites, and the cooling temperature differentials for all 61 sites.

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1.0 INTRODUCTION

The CMHC-2 (Canada Mortgage and Housing Corporation Computer Model for Heating Cost-effectively) program being developed by the Technical Research Division of CMHC, among other things, performs monthly heating requirement calculations. The program's final version will be able to perform these calculations for 61 sites in Canada, which will be representative of the entire Canadian climate.

This report presents the results of a comparison between possible mathematical models (or defining equations) for the weather data of three representative Canadian sites. The sites considered were: Ottawa, Vancouver, and Saskatoon. They represent average, mild and severe climates respectively.

The criteria used for choosing the best defining equations were:

- the equations must be comprehensive (i.e. must define the heating temperature differential for all breakeven temperatures);
- the equations must closely represent the actual weather data;
- the equations must require few defining constants;
- the equations must not require excessive computation;
- the equations must be of a form suitable for modelling similar months.

The results of this study will be used to determine the best defining equations for long term weather data to be used in the CMHC-2 program.

2.0 OBJECTIVES AND SCOPE

OBJECTIVES

The objectives of this report are:

- 1) to systematically investigate the available mathematical techniques which could be used to produce defining equations for the weather data of Ottawa, Vancouver and Saskatoon;
- 2) recommending which of these defining equations best represents the weather data to be used in the CMHC-2 program.

SCOPE

This report represents the research portion of a greater task. The greater task is the creation of a comprehensive data base for the CMHC-2 program which will allow it to model the weather data for 61 canadian sites.

It is worth noting that this report recommends a defining equation, suitable for the three sites considered. These three sites were chosen because they were representative of the 61 sites to be modelled, and hence no special problems are expected when the chosen equation (the power curve) is used to represent the weather data for the remaining 58 sites.

3.0 REQUIREMENTS OF THE MATHEMATICAL TECHNIQUE

The problem, in general terms, as mentioned in the introduction, is: "how best to model the monthly weather data, when both accuracy and simplicity are the objectives". What is meant by, "modelling the monthly weather data", is made clear in the following section.

3.1 FUNDAMENTAL PRINCIPLES

The need to define the monthly weather data arises from the use of the basic energy equation. The energy equation states that the rate at which energy (power) must be supplied to heat a dwelling is:

$$Q = HLF (\Delta T) \quad (W) \quad [1]$$

Where: Q = rate at which energy is supplied (Watts)
 $HLF = \sum UA$ = dwellings heat loss factor ($W/^{\circ}C$)
 U = heat loss coefficient for a given element ($W/m^2/^{\circ}C$)
 $\Delta T = T_{inside} - T_{outside}$ ($^{\circ}C$) (for a dwelling with no internal heat gains)

This power comes from both the furnace and the internal heat gains which includes passive solar, i.e.:

$$Q = Q_f + Q_g = HLF (T_{in} - T_{out}) \quad (W) \quad [2]$$

Where: Q_f = heating supplied by furnace (W)
 Q_g = heating supplied by internal gains (W)

The outdoor temperature for which internal heat gains just balance the heat loss through the structure is commonly referred to as the dwelling's breakeven temperature (T_{be}) and is found using equation [2], where $Q_f = 0$.

$$T_{be} = T_{inside} - \frac{Q_g}{HLF} \quad (^{\circ}C) \quad [3]$$

Using this concept of a breakeven temperature, the monthly heat loss through a building envelope is represented by:

$$E = HLF \times \left[\int_0^{1 \text{ month}} (T_{be} - T_{outside}) dt \right] \times C.F. \quad (GJ/month) \quad [4]$$

Where:

E = heat loss through envelope (GJ/month)
 $C.F.$ = dimensionless conversion constant = 8.64×10^{-5} (GJ/W.day)
 t = time (hours)

The term in square brackets represents the heating degree days contributed by one month.

The CMHC-2 program obtains a dwelling's total annual heat loss by summing the twelve monthly heat losses. This is because the breakeven temperature (T_{be}) varies with the internal gains for that month.

This justifies the need to define monthly heating degree days per month for various breakeven temperatures.

In order to find similarities amongst months which otherwise would not be possible, it was decided that monthly degree days per day would be defined, (this is equivalent to the temperature differential causing heat flow through a dwelling's envelope ($^{\circ}\text{C}$)).

Therefore equation [5] becomes:

$$E = \text{HLF} \times N \times \left[\int_0^{1 \text{ month}} (T_{be} - T_{out}) dt \right] \times \text{C.F. (GJ/month)} \quad [5]$$

Where: N = Number of days in month for which energy requirement is being calculated.

The day in equation [6] is a "synthetic" or average day.

The integration in equation [5] was performed with various breakeven temperatures using hourly weather data obtained from AES. The curves representing this integration, which describe monthly heating degree days per day (weather data) for a variable breakeven temperature are shown in Figures 1, 2 and 3 for the cities of Ottawa, Vancouver and Saskatoon respectively.

Thus, the final equation to be used within the CMHC-2 program becomes:

$$E = \text{HLF} \times N \times (\text{DD/day}) \times \text{C.F. (GJ/month)} \quad [6]$$

Where: (DD/day) = monthly heating degree days per day for a given breakeven temperature ($^{\circ}\text{C}$).

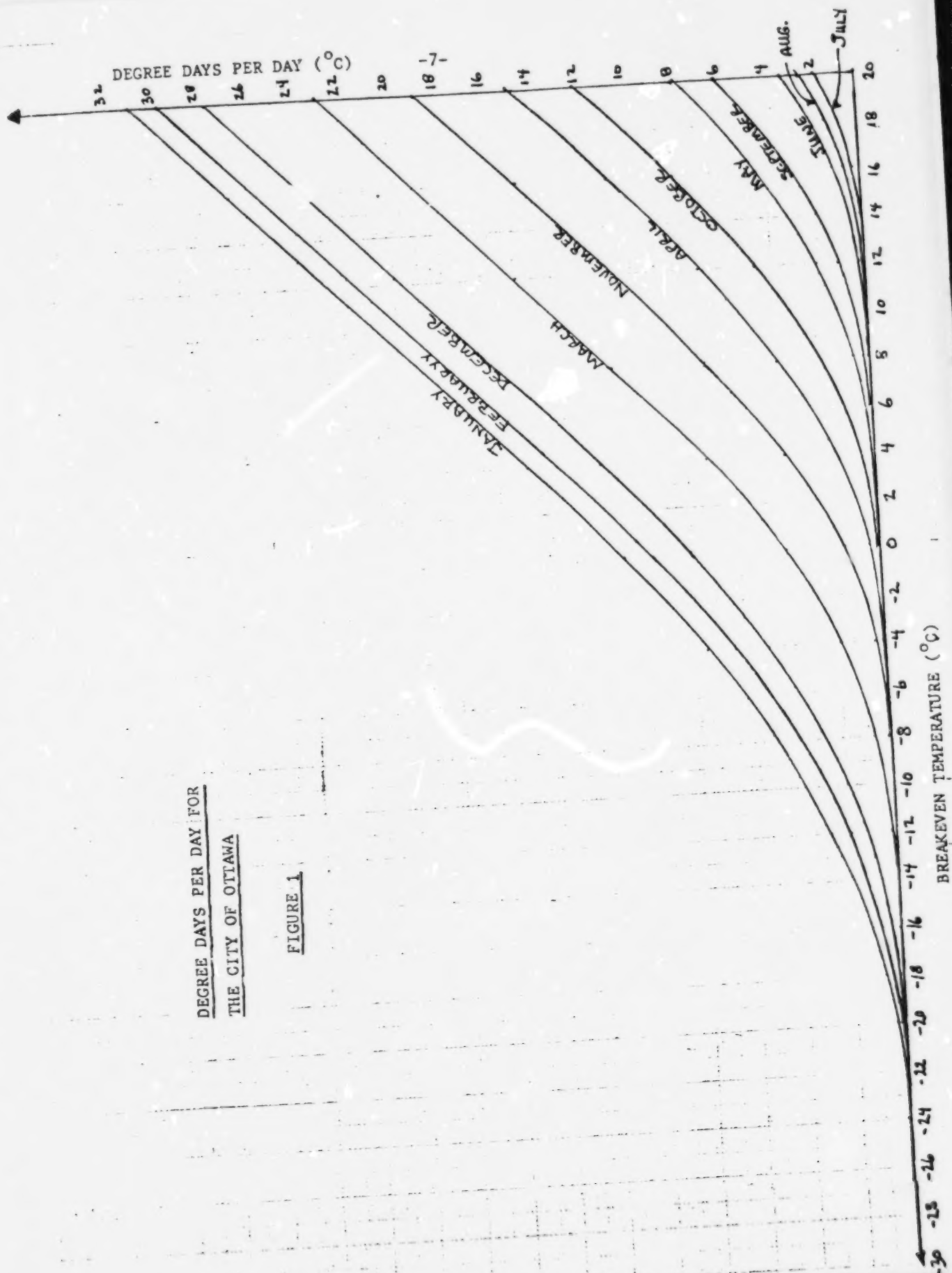
The problem treated in this report, is the choosing of an acceptable mathematical model to be used for representing this monthly degree day per day vs. breakeven temperature (weather data) curve.

From figures 1, 2, and 3 it can be seen that the fitted curve (for cold months) must transition between a horizontal line, and a 45° line. A good strategy would seem to involve fitting a curve to the non-linear portion of the weather data, and define it in such a way that the linear portion can be derived. Alternatively, if the linear portion of the chosen equation must be defined separately from the curved portion, a larger data base will be required.

The method currently used within the CMHC-2 program for defining the monthly weather data is the so called mean monthly temperature method.

DEGREE DAYS PER DAY FOR
THE CITY OF OTTAWA

FIGURE 1



DEGREE DAYS PER DAY FOR

THE CITY OF VANCOUVER

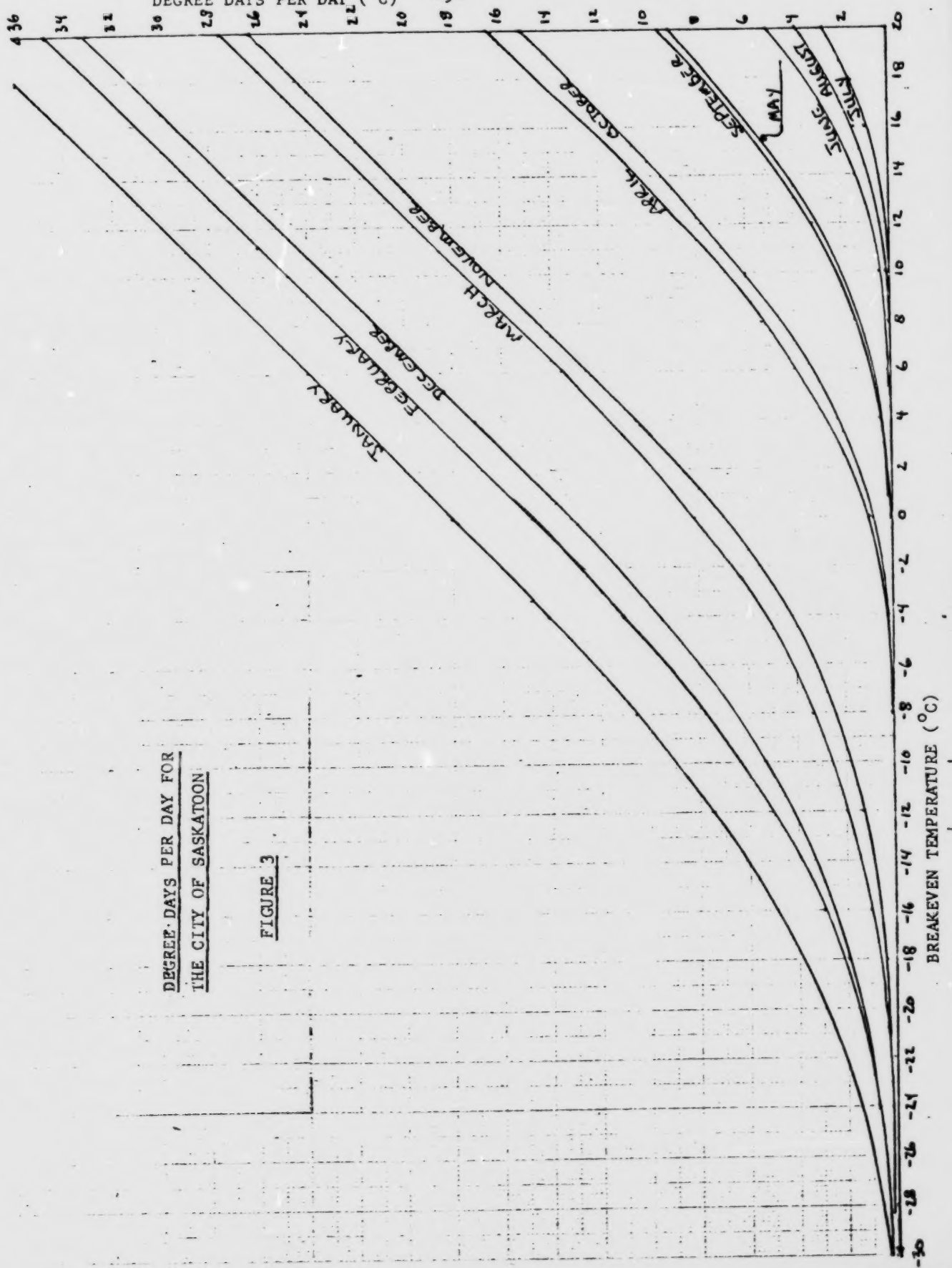
FIGURE 2

DEGREE DAYS PER DAY ($^{\circ}\text{C}$)

BREAKEVEN TEMPERATURE ($^{\circ}\text{C}$)



DEGREE DAYS PER DAY ($^{\circ}\text{C}$) -9-



DEGREE DAYS PER DAY FOR
THE CITY OF SASKATOON

FIGURE 3

With this method the monthly heat loss is:

$$E = HLF \times N \times (T_{be} - T_{mm}) \times C.F \quad (GJ/month) \quad [7]$$

Where:

T_{mm} = mean temperature for month considered ($^{\circ}C$)

The main disadvantage of this existing method is that it underestimates the heat loss when the breakeven temperature is close to the monthly mean.

3.2 REQUIRED ACCURACY OF THE MATHEMATICAL TECHNIQUE

The accuracy with which the defining equation must model the actual weather data, depends upon the effect of an error on the monthly heat loss calculation. (Recall that the model will be used once for each month after the breakeven temperature is calculated). An error may result in a loss of accuracy less than that obtained from using only a specific year of hourly weather data.

3.2.1 MAXIMUM INTERPOLATION ERROR

It was decided (for the purposes of the research portion of this project) that the maximum interpolation error should be less than or equal to 0.7 degree days per day ($^{\circ}C$). Therefore, a large percentage error in the monthly energy calculation may result if the breakeven temperature of a building is close to or less than the mean temperature of that month and the month considered is a mild one.

These errors may or may not have a significant effect on the overall space heating energy requirement.

To show this effect, a house in Ottawa with a low breakeven temperature of $10^{\circ}C$ will be considered in both January (cold month) and April (mild month).

In January there are 20.74 degree days per day at a breakeven of $10^{\circ}C$, hence an error of 0.7 will yield a percentage difference in the monthly heat flow of:

$$\frac{0.7}{20.74} \times 100 = 3.4\%$$

In April however, there are 5.49 degree days per day at this same breakeven temperature, and hence an error in monthly heat flow of 12.8%.

3.2.2 AVERAGE ERROR

This error was computed by totalling the absolute value of all the individual interpolation errors, and dividing this amount by the range over which the curve was fit. This was of significant interest when considering the error associated with using the fitted curve, although it does not take into account the fact that some interpolation errors are positive while others are negative.

During the investigation process the average error was found to be secondary of importance to the maximum interpolation error. In other words, if the fitted curve could meet the required maximum interpolation error of less than 0.7 degree days per day, it would easily meet the average error requirement which was 0.6 degree days per day.

3.2.3 AVERAGE TOTAL ERROR

The average total error was equal to the total of all interpolation errors divided by the range over which the curve was fit. This error was not used as a criteria for selecting the best method, but is of interest in a statistical sense since the error indicates to what degree the negative errors offset the positive errors.

3.3 EFFECT OF DATA BASE LIMITATIONS ON THE MATHEMATICAL TECHNIQUE

Since the CMHC-2 program will be used on a microcomputer, the maximum storage space available for the data base is limited. Therefore, the goal, as far as the data base limitations are considered, has become, "finding an acceptably accurate solution which utilizes the fewest possible defining constants and does not require excessive computation". Practically speaking this requires not only a form of equation requiring few defining constants and is easy to use, but also an equation which can model similar months. It must be remembered that while this report treats the modelling of the weather data for three cities (36 months) the CMHC-2 program must be capable of modelling 61 cities (732 months). Therefore, savings arising from an equation with few defining constants, and transferrability of that equation to similar months become important issues.

3.4 SUMMARY OF REQUIREMENTS

The requirements of the mathematical technique as presented in the previous subsections are:

- 1) The information required to define the linear portion of the weather data curve must be obtainable from the chosen equation.
- 2) The maximum interpolation and average errors must be less than 0.7 and 0.6 degree days per day respectively.

- 3) The chosen equation must require a minimum of defining constants.
- 4) Use of the equation should not require excessive computation.
- 5) The equation must be of a form suitable for modelling similar months.

4.0 ALTERNATIVE MATHEMATICAL TECHNIQUES

Many techniques have been developed to aid in modelling data using a curve. Basically, there are two approaches to the problem: firstly, if the data is considered exact (that is it does not contain statistical error) then a curve should be passed directly through the points. This is known as "interpolation". On the other hand a curve can be "fitted" to the data points if one acknowledges that there exists error in the data, either through errors in experimental measurement, or computation.

Considering the case in point, it can be said that the desired curve would be an interpolated curve so that the data would be exactly represented. This is because the only error present in the original data is the error associated with reading a thermometer and recording this reading correctly.

4.1 EXISTING METHODS

Two existing methods of representing the variable base degree day were tested for their suitability or non-suitability. These were: techniques developed for the CIRA program¹ and that developed by Lunde².

The evaluation of these two methods is presented in Appendices A.1 and A.2 respectively.

Both methods were judged to be unsuitable for use with the CMHC-2 program because of inherent inaccuracies.

4.2 INTERPOLATED CURVES

Interpolation of data points, typically requires many defining parameters. The two most prominent interpolation techniques are lagrange interpolation and the cubic spline technique. The former consists of passing a polynomial, (the only polynomial possible), through a set of data points. The cubic spline method uses cubics to interpolate between each pair of data points, and hence uses N cubics to interpolate (or model) $N + 1$ data points. This is the mathematical analogy of using a flexible drawing curve. The latter method yields excellent results but requires a very large number of defining constants, (enough to define N cubics), and hence will not be considered further. Two types of interpolated curves were investigated; the degree two polynomial and a power curve. (See Appendices A.3 and A.4 respectively). It is submitted that the interpolated power curve, has the most potential for use in the CMHC-2 program.

4.3 FITTED CURVES

Fitted curves are curves which are passed close to the data points, and a suitable error associated with having a non exact solution is minimized. Normally a least squares approach is used. This involves minimizing the total of the squared residuals (difference between observed data and the

fitted curve) associated with the fitted curve. This technique takes into account the fact that the residuals can be either positive or negative. Any type of curve can be fitted using this technique, but polynomials are the most popular.

Three possible types of fitted curves were tested. These were the degree two polynomial, the power curve and the exponential curve. (See Appendices A.5, A.6 and A.7 respectively). Neither of these models were felt appropriate for the CMHC-2 program because of inherent inaccuracies.

5.0 THE BEST SOLUTION (THE POWER CURVE)

The interpolated power curve was tested for suitability in appendix A.4, and is of the form:

$$\begin{aligned} \text{DD/day} &= A (T_{be} - Z)^B \text{ (}^\circ\text{C)} \\ \text{for: } Z &\leq T_{be} \leq T^* \end{aligned} \quad [8]$$

Where: A & B = defining constants

Z = value of T_{be} where DD/day = 0.0 (°C)

T^* = point of transition to linear portion of defining equation (°C)

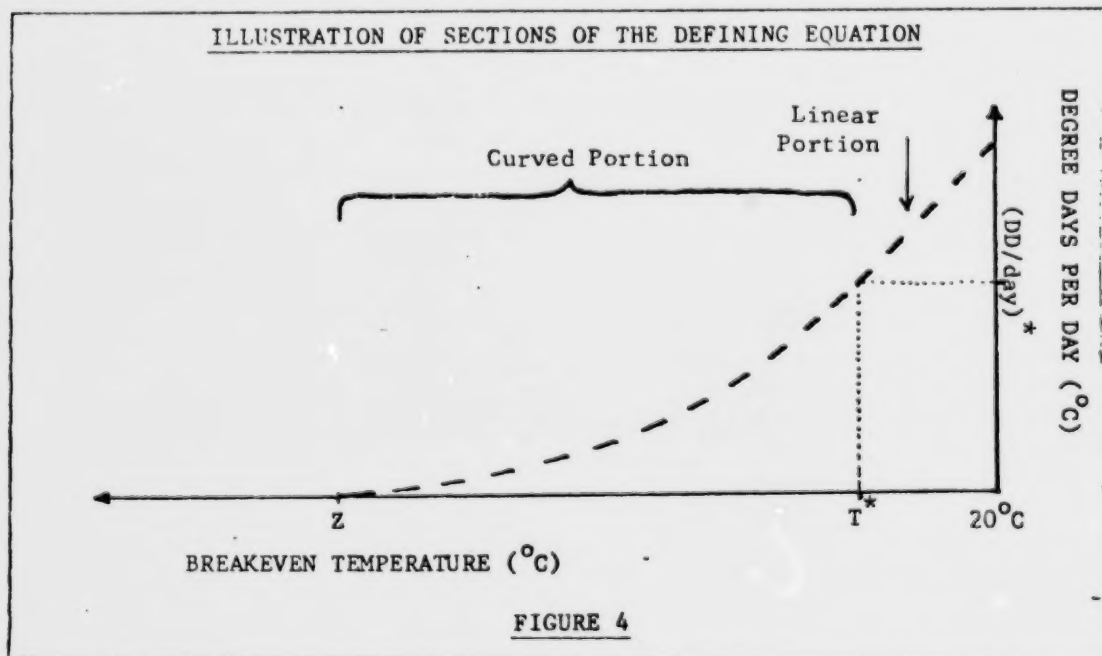
It was chosen because it best met the requirements stated in Section 3.4

The reasons for the suitability of this form of defining equation are explained in the following subsections.

5.1 HOW THE BEST POWER CURVE FOR EACH MONTH WAS FOUND

The best model for a given month is the one which minimizes the average error over the range from 20°C down to the point at which the degree days per day become zero (known as "Z"). It must be noted at this point that equation [8] models only the non linear portion of the weather data. However, (as will be shown in the following sections), the information required to define the linear portion of the curve (if one exists between 20°C and Z) can be obtained from the above equation.

The sections of the curve mentioned above are illustrated in the following figure which represents a cold month.



The best power curve equation for a given month was found using a computer program. This program selected the best point at which the linear relationship should begin. This is best explained by observing figures A4 and A5 in Appendix A. The latter figure shows a defining equation wherein the best point of transition is used, whereas the former obviously uses one of the worst. The general behaviour of the average error, and how it varied as the point of transition between the curved and linear portion of the weather data was varied, is shown in the figures which represents a typical cold month. The best point of transition is known as T^* .

Degree days per day were calculated, from the hourly data, between -50°C and $+45^{\circ}\text{C}$ for the three sites considered.

This range was required for the following reasons:

- 1) The point of transition from the curved to linear portion of the curve was varied to find the best transition point. For mild months this point of transition was in the range of 20°C to 45°C .
- 2) The point where the curved portion of the weather data curve joined the x-intercept (known as Z and equivalent to the lowest recorded temperature during the years) was near -50°C for severe months in Saskatoon.

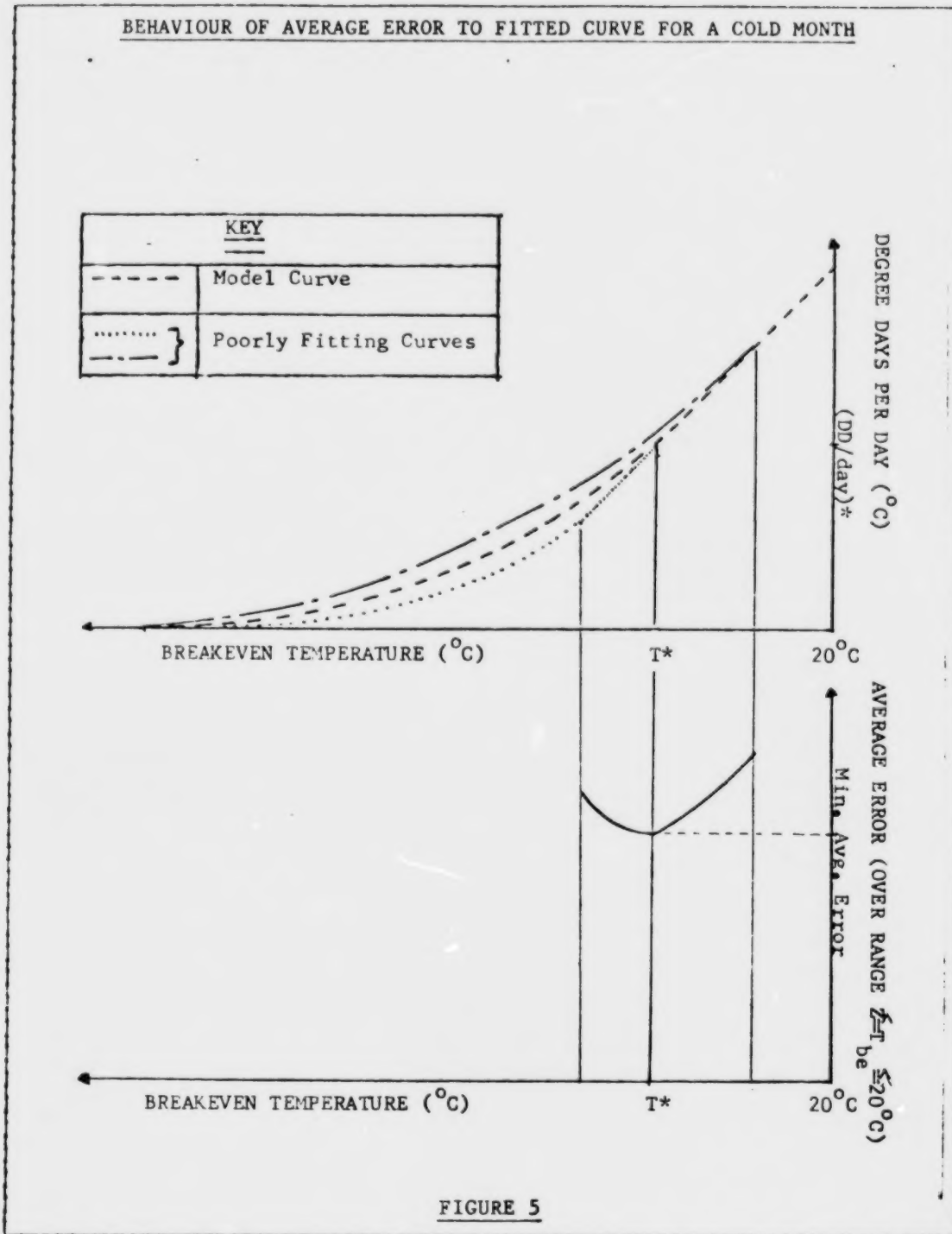
It is important to remember that the range of interest for the CMHC-2 program breakeven temperature analysis will be between -10°C and $+20^{\circ}\text{C}$.

Because of this fact, the average error for the range between Z and $+20^{\circ}\text{C}$ was minimized. This fact was noted on figure 5, and is again illustrated in figure 6, which shows the behaviour of the average error with the point of transition to the linear portion of the curve, for a warm month.

The power curve, chosen to be the best defining equation, was considered in Appendix A.4, and was treated as an interpolated curve. This term only applies, in a strict sense, to a curve which passes through two points of the actual weather data. This was the case for the curve in Figure A4. The method explained above, that of changing the transition point until the average error is minimized, is actually a fitting operation similar to at least squares method. How the best transition point was found is explained in the following subsection.

5.2 USE OF THE POWER CURVE

The best defining equation should have the capability of defining, with acceptable accuracy, both the linear and curved portions of the weather data. This requirement was noted in Section 3.4, and is the reason that the transition point must fall on the linear portion of the actual curve at a known slope.



BEHAVIOUR OF AVERAGE ERROR TO THE FITTED CURVE FOR A WARM MONTH

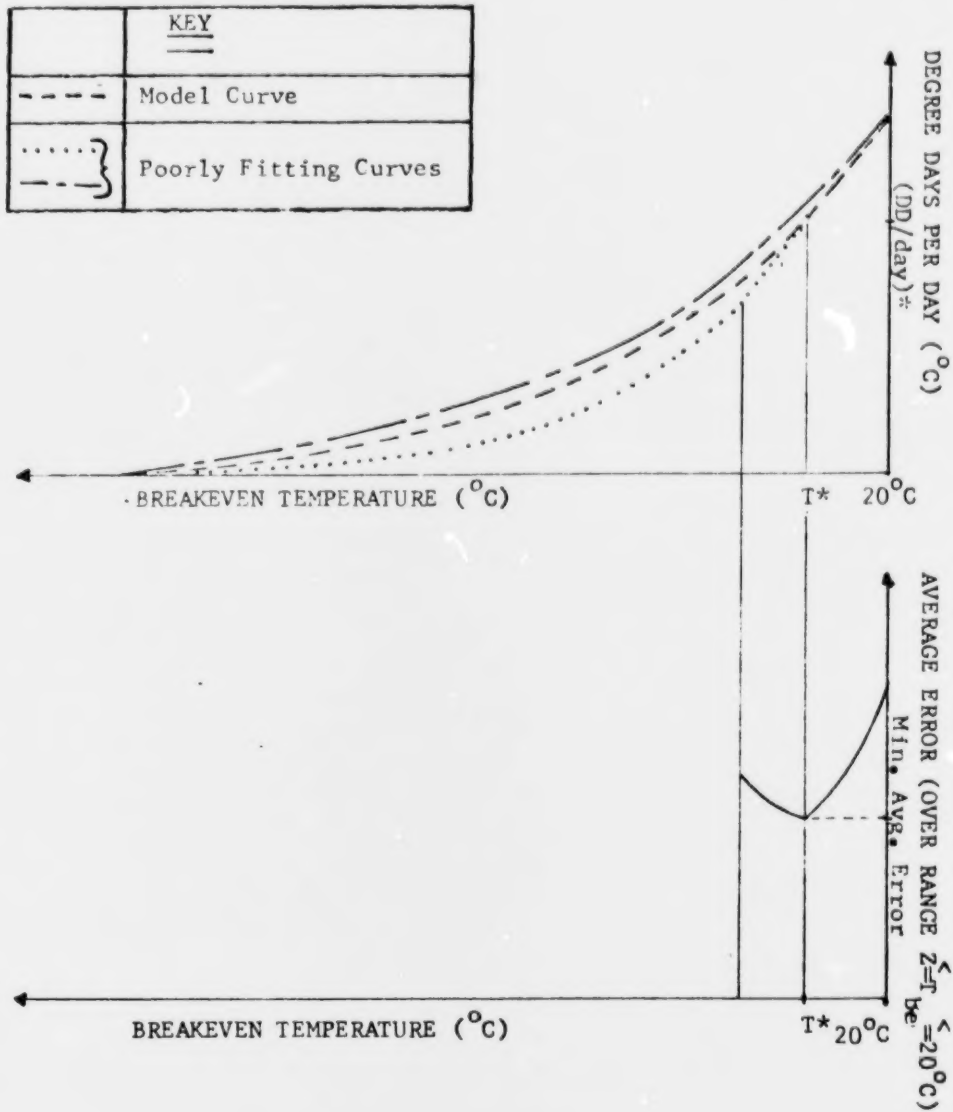


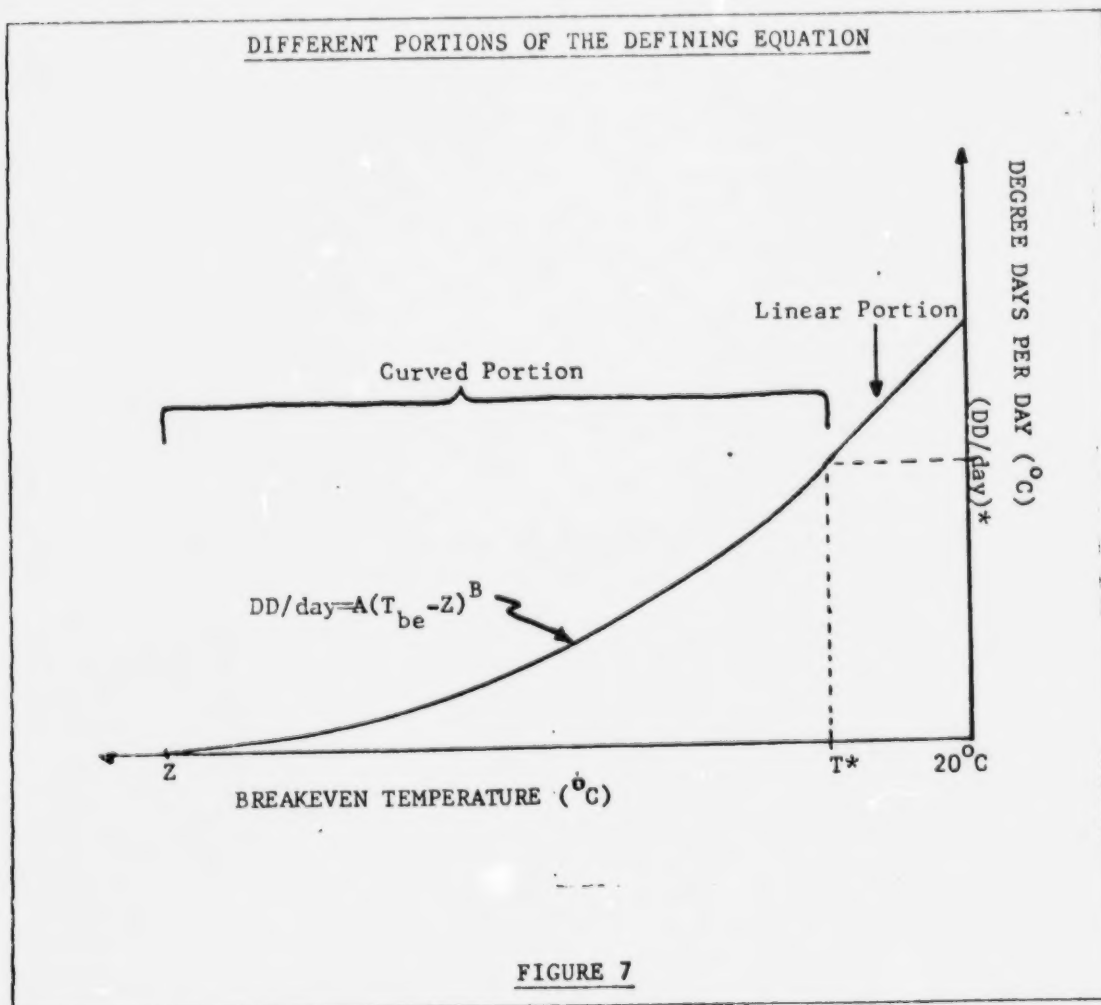
FIGURE 6

An intercepting slope of one ($M = 1$) yields an accurate and workable model. A more complicated model, having a variable intercepting slope was tested as well, however the latter method proved to be too cumbersome in light of data base limitations. The variable slope power curve is described in Appendix B, whereas the former more acceptable version is described herein as well as in Appendix A.4. A comparison of the chosen and the variable slope version of the power curves is presented in Appendix C.

The chosen version of the power curve was defined by equation [8]. i.e.:

$$\begin{aligned} \text{DD/day} &= A (T_{be} - Z)^B \quad (^\circ\text{C}) \\ \text{for: } Z &\leq T_{be} \leq T^* \end{aligned} \quad [8]$$

As noted earlier, and seen in the following figure, (applicable to cold months), equation [8] models only the curved portion of the weather data.



The way the linear portion of the curve is defined using equation [8], depends on what constraints this equation was subjected to. The power curve was required to pass through a specified point on the linear portion of the actual weather data, and meet that point with a slope of one ($M = 1$). Introducing the Z term also caused the curve to pass through the point (Z, 0.0).

The method outlined above comprises the definition of the chosen power curve. The linear portion of the fitted curve is defined using information obtained from equation [8] as explained below:

The derivative of equation [8] (the slope) is set equal to one, therefore defining the transition point to the linear portion of the defining equation. This point is known as T^* . This linear portion can be defined by:

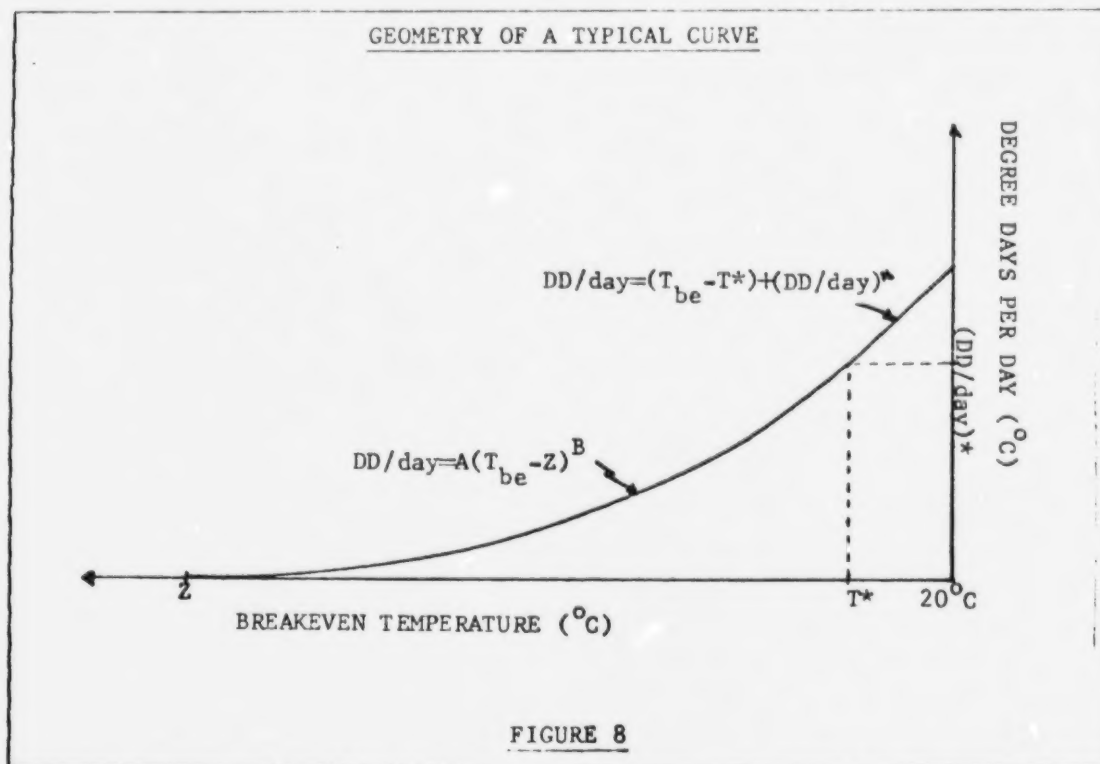
$$DD/day = M (T_{be} - T^*) + (DD/day)^* \quad [9]$$

For: $T_{be} > T^*$

Where: $M = \text{Slope} = 1$

$(DD/day)^* = \text{Number of degree days per day at } T_{be} = T^*$

The geometry which represents the above discussion is shown in figure 8:



Mathematically; T^* is equal to temperature at which $\frac{d(DD/day)}{dT} = 1$

or:

$$1 = BA (T^* - Z)^{B-1} \quad [10]$$

Solving for T^* :

$$T^* = Z + \exp [\ln (1/BA)/(B-1)] \quad (^\circ\text{C}) \quad [11]$$

$(DD/day)^*$ is the number of degree days per day when $T_{be} = T^*$, or:

$$(DD/day)^* = A(T^* - Z)^B \quad (^\circ\text{C}) \quad [12]$$

5.3 ACCURACY OF THE FITTED POWER CURVES

As noted earlier, the power curve was chosen because of its small data base requirements and its inherent precision.

A computer study was made to find the best defining equation for each month in Ottawa, Vancouver and Saskatoon. The important results of this study are shown in Table 1. It shows the maximum interpolation error, average error and average total error for each defining equation. Appendix E contains a summary of the defining constants (A, B & Z) of the power curves for Ottawa, Vancouver and Saskatoon.

Several things could be observed from the results of the computer study and Table 1:

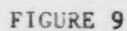
- 1) The general shape of the fitted curve, when compared to the actual data is as shown in the figure 9 (applicable to cold months).
- 2) The positive residuals tend to be negated by the negative residuals giving rise to a lower average total error than average error.
- 3) The power curve does a good job of modelling the weather data, with the top three maximum interpolation errors being 0.63, 0.56 and 0.53 $^\circ\text{C}$.

CITY	MONTH	MAXIMUM INTERPOLATION ERROR (°C)	AVERAGE ERROR (°C)	AVERAGE TOTAL ERROR (°C)
OTT.	JAN.	0.34	0.10	0.01
OTT.	FEB.	0.29	0.08	-0.01
OTT.	MAR.	0.32	0.06	-0.06
OTT.	APR.	0.56	0.19	-0.08
OTT.	MAY	0.40	0.17	0.02
OTT.	JUNE	0.21	0.07	0.03
OTT.	JULY	0.14	0.04	0.01
OTT.	AUG.	0.20	0.05	0.01
OTT.	SEPT.	0.36	0.12	-0.02
OTT.	OCT.	0.38	0.14	-0.05
OTT.	NOV.	0.41	0.11	-0.02
OTT.	DEC.	0.29	0.08	-0.01
VAN.	JAN.	0.14	0.03	0.00
VAN.	FEB.	0.14	0.04	0.02
VAN.	MAR.	0.14	0.06	0.05
VAN.	APR.	0.26	0.05	-0.04
VAN.	MAY	0.24	0.10	-0.01
VAN.	JUNE	0.33	0.11	-0.06
VAN.	JULY	0.25	0.10	0.01
VAN.	AUG.	0.28	0.10	-0.01
VAN.	SEPT.	0.26	0.06	-0.01
VAN.	OCT.	0.14	0.06	0.04
VAN.	NOV.	0.15	0.02	-0.02
VAN.	DEC.	0.16	0.04	0.01
SAS.	JAN.	0.44	0.13	-0.02
SAS.	FEB.	0.29	0.11	0.00
SAS.	MAR.	0.20	0.05	-0.02
SAS.	APR.	0.48	0.18	-0.04
SAS.	MAY	0.40	0.19	0.02
SAS.	JUNE	0.53	0.14	-0.05
SAS.	JULY	0.15	0.08	0.05
SAS.	AUG.	0.47	0.11	-0.05
SAS.	SEPT.	0.44	0.19	0.00
SAS.	OCT.	0.63	0.17	-0.08
SAS.	NOV.	0.23	0.04	-0.02
SAS.	DEC.	0.10	0.03	0.01

TABLE 1

RESULTS OF COMPUTER ANALYSIS TO FIND BEST
POWER CURVE FOR OTTAWA, VANCOUVER AND
SASKATOON

NOTE: See Section 3.2 for a definition of the types of accuracy.



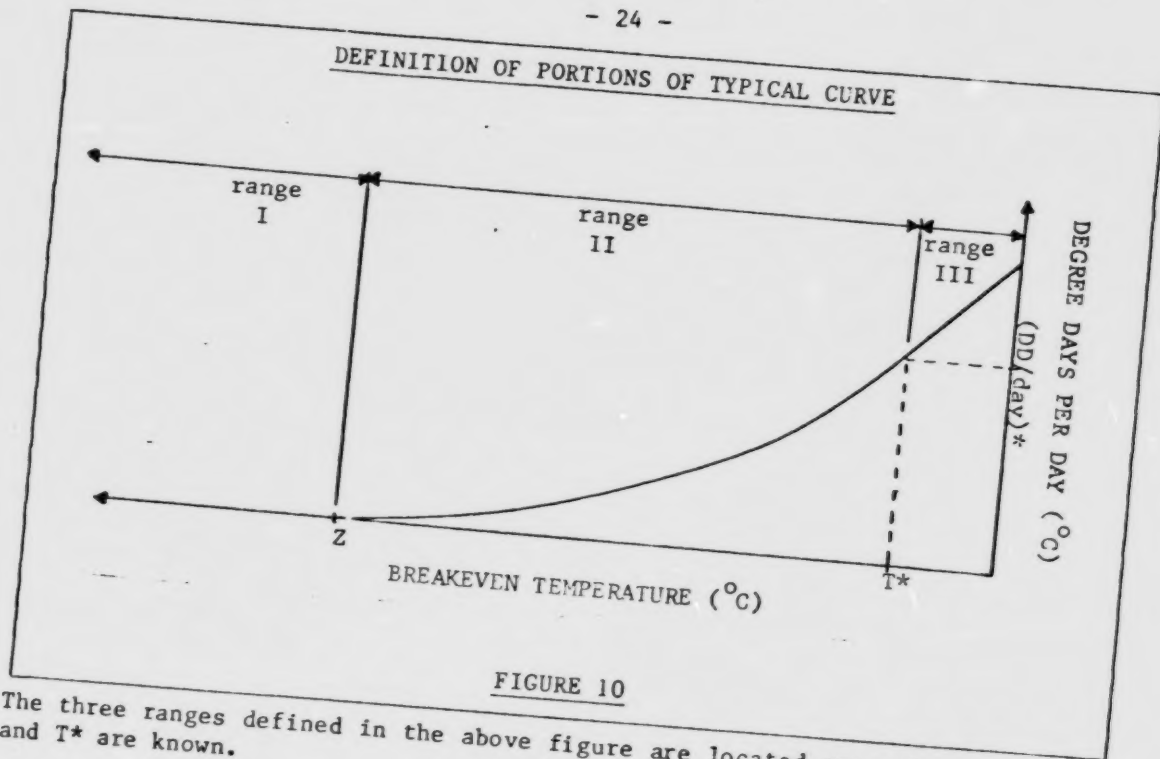
5.4 RECOMMENDED USE OF THE POWER CURVE IN CMHC-2

The defining equation for the power curve was given as:

$$\text{DD/day} = A(T_{be} - Z)^B \quad (^\circ\text{C}) \quad [8]$$

For: $Z \leq T_{be} \leq T^*$

The CMHC-2 program will first compute the monthly breakeven temperature using equation [3]. Then, equation [8] will be used to find the number of degree days per day at that breakeven temperature. In order to do this, the portion of the curve in which T_{be} lies, must be determined. The portions of the curve are shown in Figure 10.



The three ranges defined in the above figure are located once the constants Z and T^* are known.

T^* was defined earlier as:

$$T^* = Z + \exp [\ln (1/BA)/(B-1)] \quad (^\circ\text{C}) \quad [11]$$

If T_{be} falls in range I ($T_{be} < Z$) the solution is obviously: $DD/day = 0.0$.

If T_{be} falls in range II ($Z \leq T_{be} \leq T^*$) then the solution is:

$$DD/day = A(T_{be} - Z)^B \quad (^\circ\text{C}) \quad [8]$$

In range III ($T_{be} > T^*$) equation [10] is used:

$$DD/day = (T_{be} - T^*) + (DD/day)^* \quad (^\circ\text{C}) \quad [9]$$

or in summary form:

Range in which T_{be} falls ($^\circ\text{C}$)	Number of Degree Days Per Day ($^\circ\text{C}$)
$T_{be} < Z$	$DD/day = 0.0$
$Z \leq T_{be} \leq T^*$	$DD/day = A(T_{be} - Z)^B$
$T_{be} > T^*$	$DD/day = (T_{be} - T^*) + (DD/day)^*$

It can be seen that the power curve is simple to use. The actual method used by CMHC-2 is suggested in Figure 11.

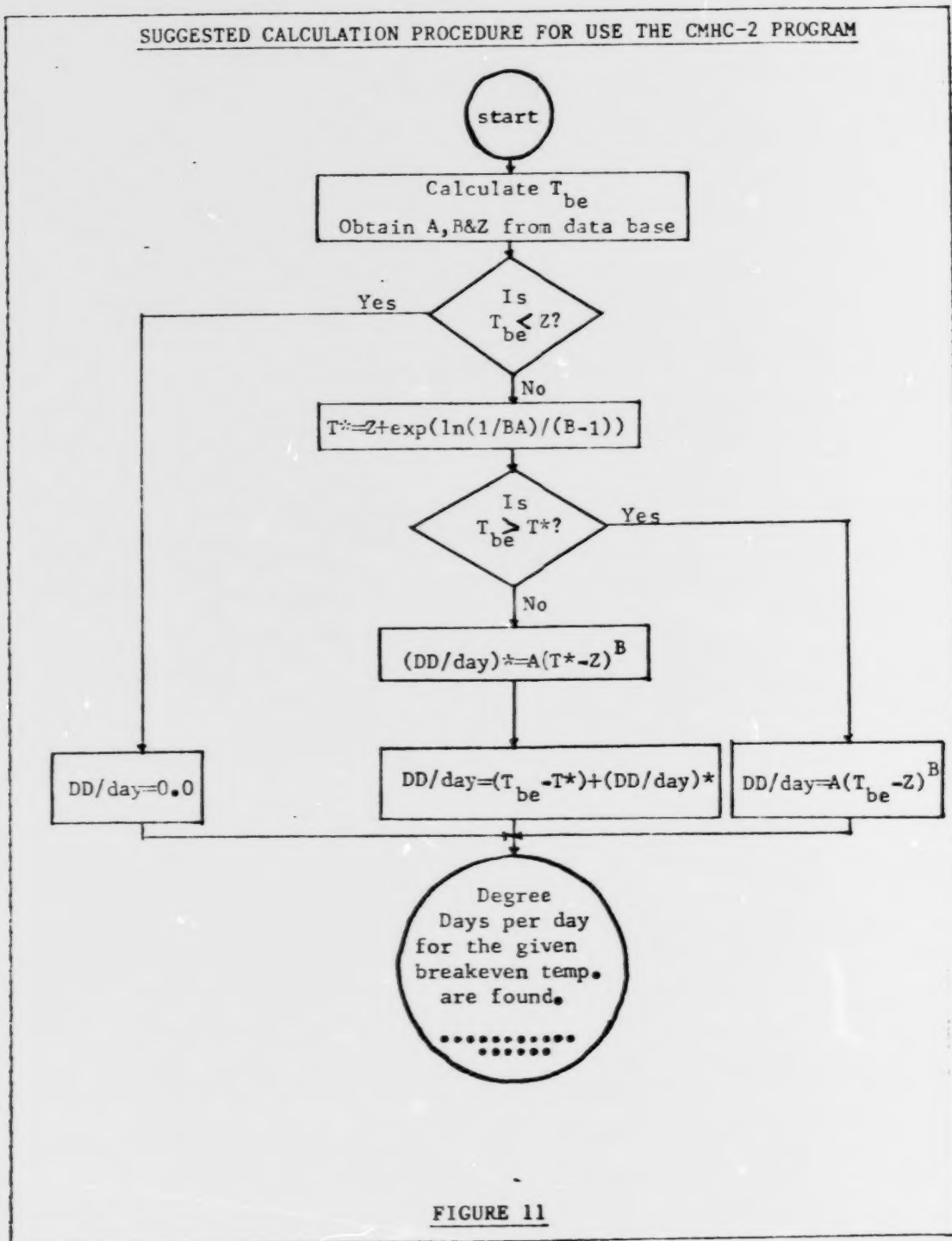


FIGURE 11

6.0 INVESTIGATION OF POSSIBLE SAVINGS IN THE REQUIRED DATA BASE SIZE

Having concluded that the power curve is an acceptable defining equation for the problem considered, ways were sought to minimize the size of the data base required to define these equations.

Consideration of equation [8] and figures 1, 2, & 3 shows that the Z term is unique for each month. Physically, this constant is the lowest temperature recorded for a given month during the years in which the hourly data was collected. Since this constant is both unique for each month, and is an integer, savings in the data base size are not possible through either simplifying it (i.e. having fewer significant figures) or through its transferrability to other months having the same constant.

However, it was felt that savings in the overall data base size were possible, through both simplifying and investigating the transferrability of the A & B constants in equation [8]. Thus the following studies were carried out:

- 1) A computer aided study was made to determine the least number of significant figures required to define the A & B constants for a given month, while an accurate model was still maintained.
- 2) The computer was again used to discover if it was possible to use the A & B constants, which define the power curve for a given month, to model other month's weather data. And if transferrability was practical, how many significant figures were required in the defining constants in order to obtain an acceptable model.

The details of the above two studies are presented in Appendix F. The least number of significant figures practical was found to be four, while a great many "matches" between similar months were also found.

7.0 INVESTIGATION OF POSSIBLE CORRELATION BETWEEN ANY TWO SIMILAR MONTH'S
MEAN MONTHLY TEMPERATURES

Savings in the computing costs of future work were considered possible, if a relationship could be found between the mean monthly temperatures of any two months with similar defining equations (i.e. months with transferable A & B constants).

Appendix G presents the correlations found between the differences in two month's mean monthly temperatures, and the similarity of their defining equations.

The results of this study were dissapointing since it was discovered that the curves of two months could have similar shapes, (hence, defining constants), but widely different mean monthly temperatures.

8.0 CONCLUSIONS

- 1) Neither the method used in the CIRA program or that published by Lunde are suitable for representing the weather data in the CMHC-2 program.
- 2) The power curve is the best defining equation for the weather data because it is precise, concise and comprehensive.
- 3) The largest maximum interpolation error between the defining equation and the actual data for the three sites tested using defining constants of seven significant figures, was 0.63 degree days per day.
- 4) The number of significant figures in the A&B constants for a given month can be limited to four. For the three sites tested the largest maximum interpolation error was 0.64 degree days per day when four significant figures were used to define the A & B constants.
- 5) The largest maximum interpolation error expected for the 61 sites to be modelled is 0.8 degree days per day.
- 6) Approximately 50% of the defining constants (A&B) are redundant for the three cities considered. This translates into a need for a data base containing approximately 1300 constants, rather than 2196 if transferrability were not considered, for the CMHC-2 program. (This includes a Z integer constant for each month.)

9.0 RECOMMENDATIONS

9.1 GENERAL RECOMMENDATIONS

- 1) That the power curve be used to model the weather data in the CMHC-2 program, (with four significant figures specifying the A & B constants).
- 2) That a program be developed that will produce the defining equations from the hourly temperature data tape supplied by AES, for the test sites of Ottawa, Vancouver and Saskatoon (the logic of such a program was shown in Figure 12).
- 3) That the tape for the hourly dry bulb temperatures be obtained from AES for the remainder of the 61 Canadian sites needed for the CMHC-2 program. The computer program from step 2 should be used to obtain the defining equations for each month's weather data.
- 4) That steps 2 and 3 be repeated to develop cooling profiles with dry bulb data (except for some cities in the far north.)
- 5) That steps 2 and 3 be repeated to develop heating profiles using wet bulb data.
- 6) That steps 2 and 3 be repeated to develop cooling profiles using wet bulb data (except for some cities in the far north.)

FLOWCHART OF RECOMMENDED METHOD
FOR OBTAINING DATA BASE FOR CMHC-2 (COOLING - DRY BULB)

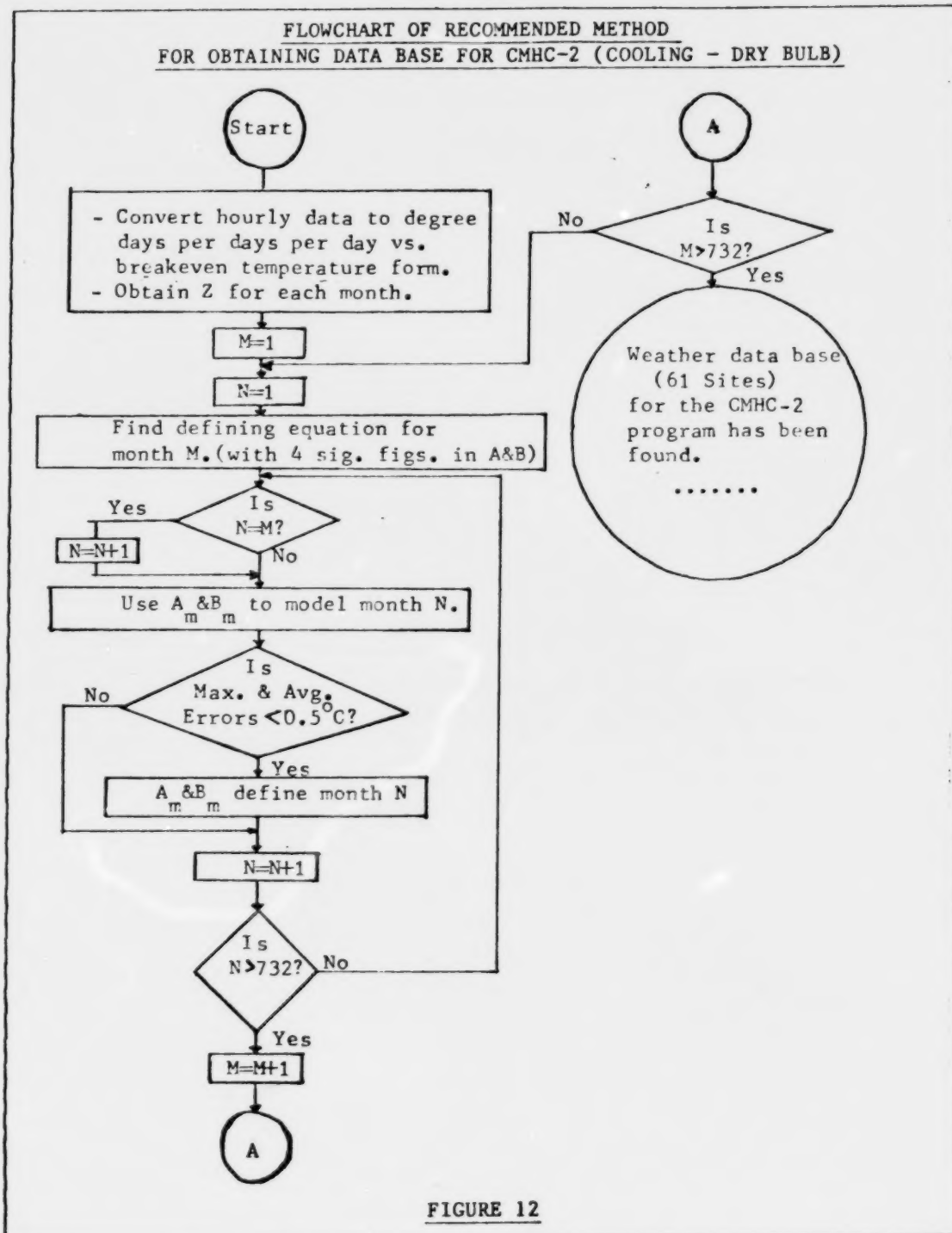


FIGURE 12

9.2 EXPECTED FUTURE DEVELOPMENT COSTS

The following projected costs concern the development of the data base for CMHC-2:

RECOMMENDATION	EXPLANATION	EXPECTED COST (\$)
2	Develop and debug program to convert hourly data to defining equations.	2,000.00
3	Develop profiles for heating (dry bulb).	1,500.00
4	Develop profiles for cooling (dry bulb).	2,500.00
5	Develop profiles for heating (wet bulb).	1,750.00
6	Develop profiles for cooling (wet bulb).	1,750.00
	Report on technique and results.	3,000.00
	Incorporate relationships in CMHC-2 Program.	500.00
	Allowance for editorial and visual work associated with the report.	2,000.00
	TOTAL	15,000.00

10.0 REFERENCES

1. Sonderegger, Garnier & Dixon, Computerized Instrumented Residential Analysis (CIRA), University of California, November 1981, Draft, Section 7, pages 32, 33.
2. Peter Lunde, Solar Thermal Engineering, John Wiley and Sons, 1980, pages 20 to 27.
3. Burden, Faires and Reynolds, Numerical Analysis, Prindle, Weber and Schmidt, 1978, pages 136 - 145.

APPENDIX A

POSSIBLE MATHEMATICAL MODELS

<u>APPENDIX</u>	<u>CONTENTS</u>	<u>PAGE</u>
A.1	VARIABLE BASE DEGREE DAYS (CIRA)	
A.2	VARIABLE BASE DEGREE DAYS (LUNDE)	
A.3	INTERPOLATED POLYNOMIAL OF DEGREE TWO	
A.4	INTERPOLATED POWER CURVE	
A.5	FITTED POLYNOMIAL OF DEGREE TWO	
A.6	FITTED POWER CURVE	
A.7	FITTED EXPONENTIAL CURVE	

A.1 CIRA VARIABLE BASE DEGREE DAYS¹

The equation used by the CIRA program, (Computerized Instrumented Residential Analysis is:

$$D_{h,c}^{d,n} = \frac{1}{2} \left([\Delta T] + \mu_{h,c}^{d,n} [\delta_{h,c}^{d,n} - [\Delta T] + 1]_{h,c}^{d,n} \right) \quad (^\circ\text{C}) \quad [A1]$$

where:

$$\Delta T = \text{IND}_{h,c} (T_{a,n}^d - T_{f,n}^d) \quad (^\circ\text{C}) \quad [\text{A2}]$$

$D_{h,c}^{d,n}$ = degree days per day ($^{\circ}\text{C}$) for either day or night, heating or cooling.

IND_{h,c} = seasonal index (+1 for heating, -1 for cooling).

$$\delta_{h,c}^{d,n}, \mu_{h,c}^{d,n}, i_{h,c}^{d,n} \quad \text{are empirical degree day coefficients.}$$

$[X]^+$ is equal to X , if $X \geq 0$, zero otherwise.

$T_{a^d, n}$ = average indoor temperature ($^{\circ}\text{C}$).

$$T_f^{d,n} = \text{monthly average temperature (}^\circ\text{C)}.$$

A partial list of coefficients is contained in Table A1.

It should be noted that equation [A2] represents the degree days per day calculated by the mean monthly temperature method, (i.e the average indoor temperature minus the average monthly outdoor temperature). It can therefore be seen that equation [A1] represents degree days per day which are calculated by the mean monthly temperature method. The method contains correction terms which probably force it to agree with some empirical data. An example will illustrate the use of this method and the effect of the correction terms:

		COEFFICIENTS FOR USE WITH CIRA PROGRAM					
CITY:		$\mu_{h,c}^d$	$\mu_{h,c}^n$	$i_{h,c}^d$	$i_{h,c}^n$	$\delta_{h,c}^d$	$\delta_{h,c}^n$
WASHINGTON	heating	1.13266 E-05	2.07281 E-07	3.90807	4.79756	13.7	17.8
	cooling	9.3201 E-04	4.98985 E-03	2.75180	2.4076	11.1	7.8
SAN FRANCISCO	heating	3.23912 E-06	6.75566 E-03	4.61008	2.53399	10.0	5.0
	cooling	1.82388 E-12	1.13769 E-16	8.12677	10.3759	16.7	19.4
ALBUQUERQUE	heating	8.60275 E-04	6.61429 E-04	3.17141	2.85504	8.3	11.7
	cooling	4.38589 E-03	3.92858 E-03	2.30562	2.49328	11.1	7.8
MINNEAPOLIS	heating	3.30545 E-02	1.14680 E-04	1.77976	3.13452	8.9	16.7
	cooling	4.71026 E-04	7.42717 E-02	2.80824	1.60817	14.8	5.7

TABLE A1

QUESTION: Using the CIRA variable base degree days, calculate the nighttime cooling degree days per day in Minneapolis, if:

$$\begin{aligned}T_a^n &= 21^\circ\text{C} \\ T_f^n &= 23^\circ\text{C}\end{aligned}$$

ANSWER: Equation [A2] gives: $\Delta T = -1 (21 - 23) = 2^\circ\text{C}$

$$\begin{aligned}DD_c^n &= \frac{1}{4} (2 + 7.42717 \times 10^{-2} [5.7 - (21 - 23)]^{1.60817}) \\ &= 1.99^\circ\text{C}\end{aligned}$$

If the simple mean monthly temperature method were used the number of degree days per day becomes:

$$\begin{aligned}DD/\text{day} &= IND \times K \times (T_a - T_f) \\ &= -1 \times 12/24 \times (21 - 23) \\ &= 1^\circ\text{C}\end{aligned}$$

Where:

K = multiplying constant representing contribution of nighttime cooling to the cooling for the whole day (hr./hr.).

A.2 VARIABLE BASE DEGREE DAYS (LUNDE)²

The proposed equation for monthly degree days, which incorporates a variable base, as put forth by Lunde is:

$$DD_m = (N_m \Delta T_b [0.744 + 0.00387 D_a - \frac{0.5 D_a^2}{10^6}] \times N_m \exp - \frac{\Delta T_b + 11.11}{9.02}) \quad (^\circ\text{C}) \quad [A3]$$

where: DD_m = monthly degree days ($^\circ\text{C days}$)

N_m = number of days in month considered

$\Delta T_b = (T_{be} - T_{mm})$ = temp. base to average temp. difference ($^\circ\text{C}$)

D_a = total annual degree days to the base 18.33°C ($^\circ\text{C days}$)

Lunde notes that the equation is valid for locations with annual degree days between 1000 & 4000 ($^\circ\text{C Days}$). He also notes that the equation was "derived from the study of degree days at a number of locations".

To check the validity or non-validity of this formula, a plot of monthly degree days versus breakeven temperature is shown in Figure A1 for Ottawa - January, where:

$$N_m = 31$$

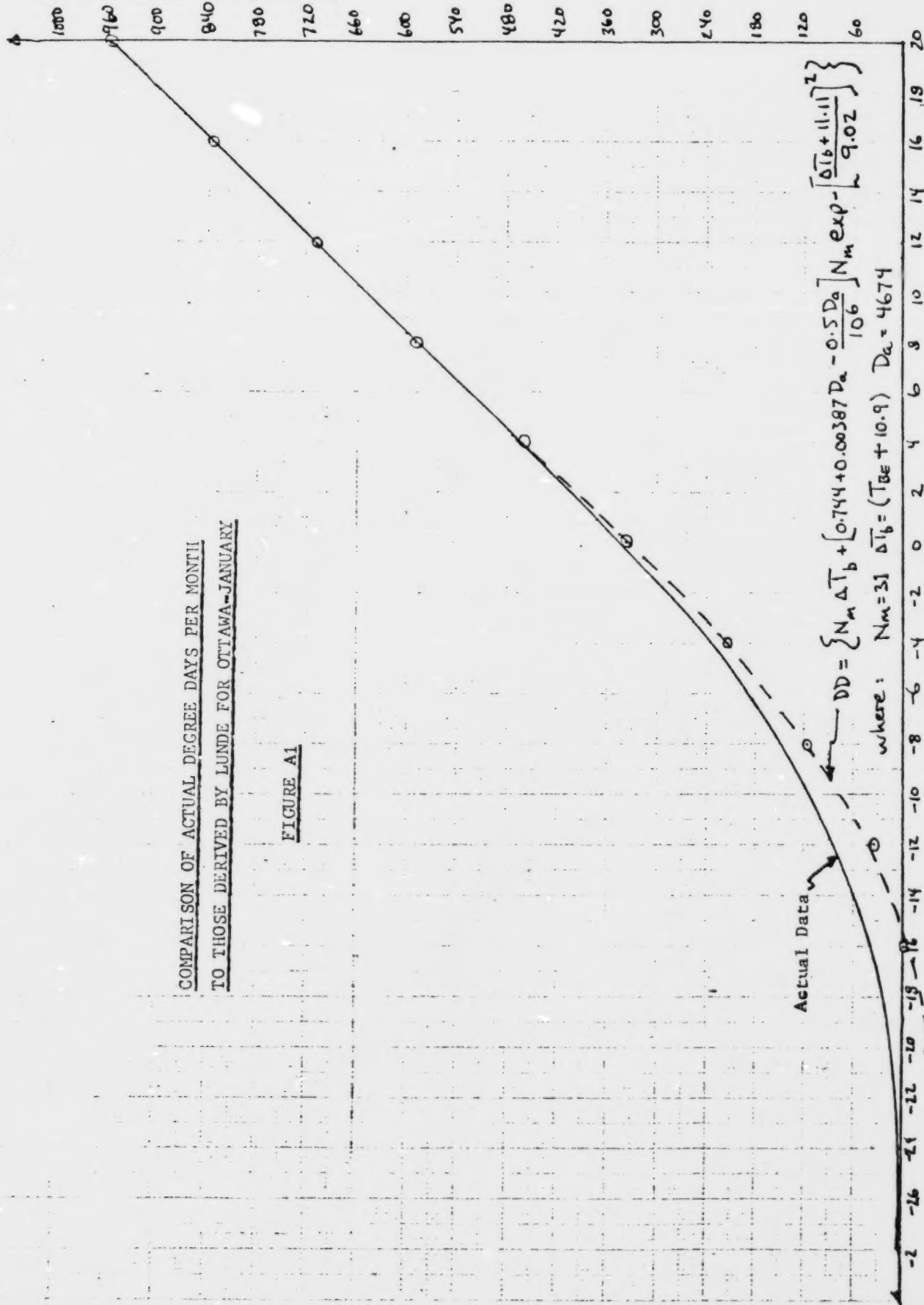
$$\Delta T_b = (T_{be} + 10.9) \quad ^\circ\text{C}$$

$$DD_a = 4674 \quad (^\circ\text{C Days})$$

It can be noted from Figure A1 that poor agreement with the actual data is found for low breakeven temperatures. Similar disagreements were found for Vancouver - January where the annual degree days fall within the specified range.

No hints are provided by Lunde as to how the equation was derived, but it is the opinion of the author that such an approach will not meet the accuracy requirements of Section 3.2.

MONTHLY DEGREE DAYS ($^{\circ}\text{C}\cdot\text{DAYS}$) - - - -37-



COMPARISON OF ACTUAL DEGREE DAYS PER MONTH
TO THOSE DERIVED BY LUNDE FOR OTTAWA-JANUARY

FIGURE A1

A.3 INTERPOLATED POLYNOMIAL OF DEGREE TWO

As noted in Section 4.2, a polynomial known as a lagrange polynomial can be passed through any number of points. The order of this polynomial is one less than the number of points it passes through. Thus, a second order polynomial, (which requires three constants to define), will pass through any three points.

In considering the acceptability of such a curve, the data for Ottawa - March was used as a test case. A good solution was judged to be so, based on the maximum interpolation error, average error and data base requirements as well as transferability considerations. A second order lagrange polynomial has the form:

$$\begin{aligned} \text{DD/day} = & (\text{DD/day})_1 \times \frac{(T_{be} - T_2)(T_{be} - T_3)}{(T_1 - T_2)(T_1 - T_3)} + (\text{DD/day})_2 \times \frac{(T_{be} - T_1)(T_{be} - T_3)}{(T_2 - T_1)(T_2 - T_3)} \\ & + (\text{DD/day})_3 \times \frac{(T_{be} - T_1)(T_{be} - T_2)}{(T_3 - T_1)(T_3 - T_2)} \quad (^\circ\text{C}) \end{aligned} \quad [\text{A4}]$$

when passed through the points $(T_1, (\text{DD/day})_1)$, $(T_2, (\text{DD/day})_2)$ and $(T_3, (\text{DD/day})_3)$.

The results of fitting a second order polynomial to the non-linear portion of the weather data for Ottawa - March is shown in figure A2. The curve was fitted between the point where the actual data met the breakeven temperature axis, and where it first became linear, (that is between Z & T_0).

The points through which the equation was passed are:

$T_{be} (^\circ\text{C})$	DD/day ($^\circ\text{C}$)
15	17.93
- 4	1.85
-23	0.00

Equation [A4] becomes:

$$\text{DD/day} = \frac{17.93 (T_{be}+4)(T_{be}+23)}{(15+4)(15+23)} + \frac{1.85 (T_{be}-15)(T_{be}+23)}{(-4-15)(-4+23)}$$

$$\text{or: DD/day} = (1.97091 \text{ E-02 } T_{be}^2) + (6.29515 \text{ E-01 } T_{be}) + 4.05271 \quad [\text{A5}]$$

From figure A2 it can be seen that the maximum interpolation error is greater than the specified maximum of 0.7°C . In addition the data base would have to contain three "real" defining constants for each curve. Additional problems with this method also arise when one tries to define the linear portion of the curve using equation [A5]. This problem is best explained by the fact that, although the interpolated curve passes through a point on the linear portion of the data, its slope at that point was not specified and is

therefore not obtainable from equation [A5]. Because of this shortcoming, a larger data base would be required to define the linear portion of the curve.

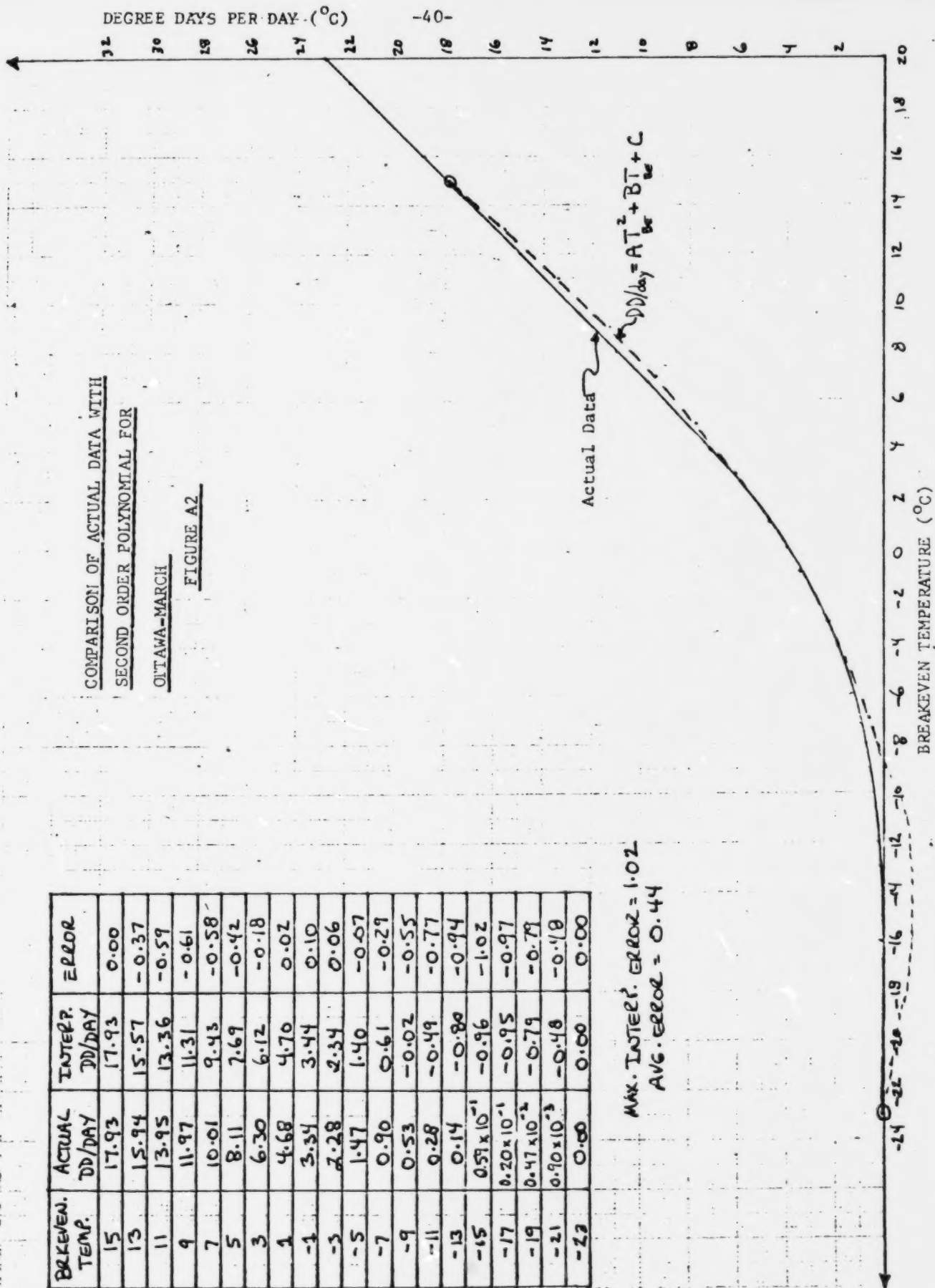
Finally, the equation is unique to the curve for which it was computed, and hence has poor transferability potential.

BREKVEN. TEMP.	ACTUAL DD/DAY	INTERP. DD/DAY	ERROR
15	17.93	17.93	0.00
13	15.94	15.57	-0.37
11	13.95	13.36	-0.59
9	11.97	11.31	-0.66
7	10.01	9.43	-0.58
5	8.11	7.69	-0.42
3	6.30	6.12	-0.18
1	4.68	4.70	0.02
-1	3.34	3.44	0.10
-3	2.28	2.34	0.06
-5	1.47	1.40	-0.07
-7	0.90	0.61	-0.29
-9	0.53	-0.02	-0.55
-11	0.28	-0.49	-0.77
-13	0.14	-0.80	-0.94
-15	0.59×10^{-1}	-0.96	-1.02
-17	0.20×10^{-1}	-0.95	-0.97
-19	0.47×10^{-2}	-0.79	-0.79
-21	0.90×10^{-3}	-0.48	-0.48
-23	0.00	0.00	0.00

MAX. INTERP. ERROR = 1.02
AVG. ERROR = 0.44

COMPARISON OF ACTUAL DATA WITH
SECOND ORDER POLYNOMIAL FOR
OTTAWA-MARCH

FIGURE A2



A.4 INTERPOLATED POWER CURVE

A simple power curve would be of the form:

$$DD/day = A T_{be}^B (^\circ C) \quad [A6]$$

Where A&B are constants that are defined by the required shape of the curve. Difficulty in using this general form arises when either non-integer values for B, or negative values for T_{be} are used. Because of this, a shift term "Z" was also used, making the workable equation:

$$DD/day = A (T_{be} - Z)^B (^\circ C) \quad [A7]$$

Where Z is the value of T_{be} where the actual weather data curve meets the zero degree day per day axis. Introducing the shift term also has the advantage of forcing the curve to pass through the point (Z,0.0). There are several ways to define the constants A&B, but in this case they are defined by; specifying a point on the linear portion of the data as being a point on the power curve, and defining the slope at that point. The point at which the weather data first becomes linear is known as $(T_o, (DD/day)_o)$ and the specified slope is (M).

Mathematically:

$$\frac{d(DD/day)}{dT_{be}} = M = BA (T_o - Z)^{B-1} (^\circ C) \quad [A8]$$

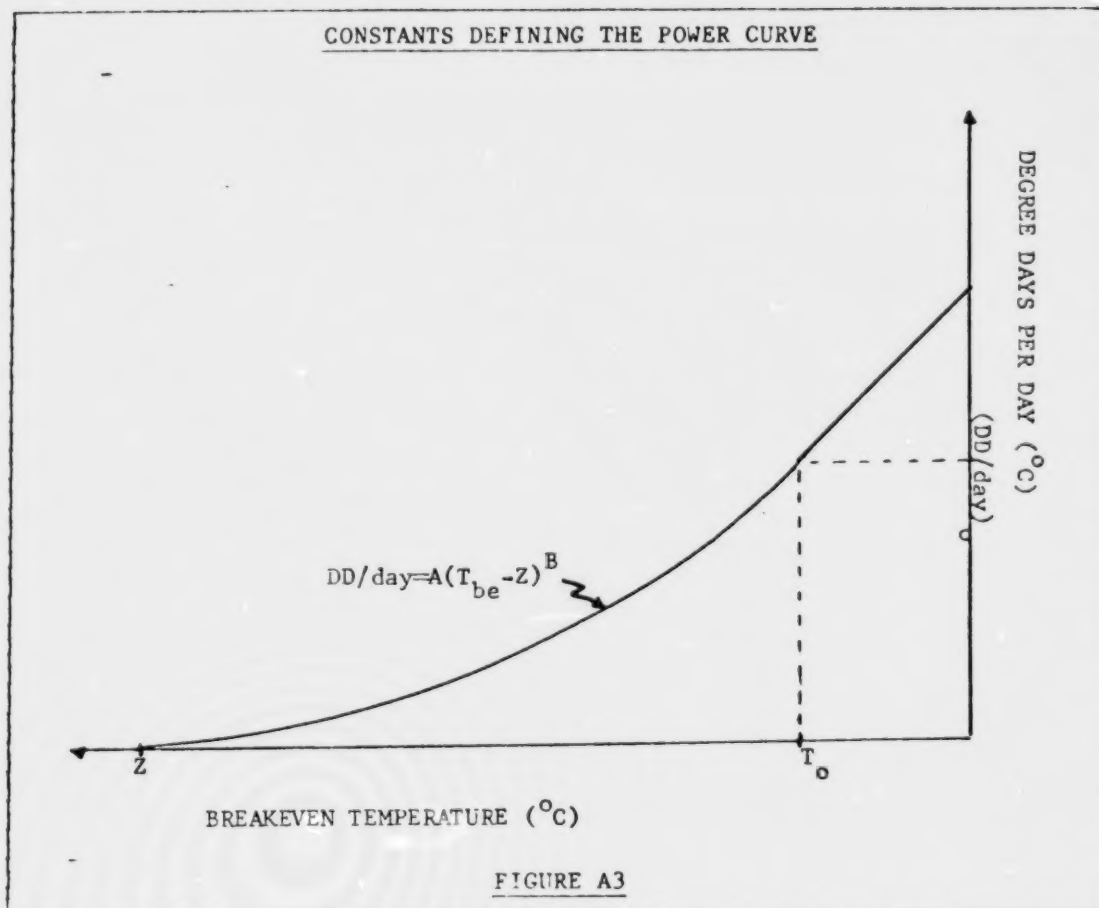
$$\text{Also: } (DD/day)_o = A (T_o - Z)^B (^\circ C) \quad [A9]$$

Solving these two equations for A&B we find:

$$B = \frac{M (T_o - Z)}{(DD/day)_o} \quad [A10]$$

$$A = (DD/day)_o (T_o - Z)^{-B}$$

The following figure shows the terms defined above:



An example illustrates the use of this method: Consider Ottawa - March and let the slope at which the power curve joins the straight line equal one, (i.e. $M = 1$).

here: $T_o = 15^\circ\text{C}$
 $(DD/day)_o = 17.93^\circ\text{C}$
 $Z = -23^\circ\text{C}$

From equation [A10]: $B = \frac{15 + 23}{17.93} = 2.11935$

From equation [A11]: $A = 17.93 (15 + 23)^B = 8.04380 \times 10^{-3}$

Therefore equation [A7] becomes:

$$DD/day = 8.04380 \text{ E-03 } (T_{be} + 23)^{2.11935} \text{ } (^\circ\text{C})$$

Figure A4 shows a comparison of the fitted curve to the actual data. It can be seen that poor agreement with the actual data is found.

However, if the point at which the fitted curve transitions to the linear portion of the fitted curve, is moved in the direction down the extended linear portion of the actual data, both the maximum interpolation error and average errors fall below the established maximums. For example; see figure A5 which shows the best point of transition between the curved and linear portions of the curve. The best transition point, (known as $(T^*, (DD/day)^*)$), is $(4^\circ\text{C}, 7.18^\circ\text{C})$. It was found by a computer program which minimized the sum of the absolute values of all residuals, between the fitted curve and the actual data. (Equivalent to minimizing the average error). The fact that this optimization worked is shown by figure A5, where the maximum interpolation error and average errors were 0.32 and 0.06°C respectively.

The chosen power curve requires three defining constants (A, B & Z), however Z is an integer. The potential transferability of the constants to other similar months is evident from the form of the power curve. This aspect of the analysis was shown in Section 6.0.

BREKEVEN. TEMP.	ACTUAL DD/DAY	INTERP. DD/DAY	ERROR
15	17.93	17.93	0.00
13	15.94	15.99	0.05
11	13.95	14.16	0.21
9	11.97	12.46	0.49
7	10.01	10.86	0.85
5	8.11	9.39	1.28
3	6.30	8.02	1.72
1	4.68	6.77	2.09
-1	3.34	5.63	2.29
-3	2.28	4.60	2.32
-5	1.47	3.68	2.21
-7	0.90	2.87	1.97
-9	0.53	2.16	1.63
-11	0.28	1.56	1.28
-13	0.14	1.06	0.92
-15	0.59×10^{-1}	0.66	0.60
-17	0.20×10^{-1}	0.36	0.34
-19	0.47×10^{-2}	0.15	0.15
-21	0.90×10^{-3}	0.35×10^{-1}	0.34×10^{-1}
-23	0.00	0.00	0.00

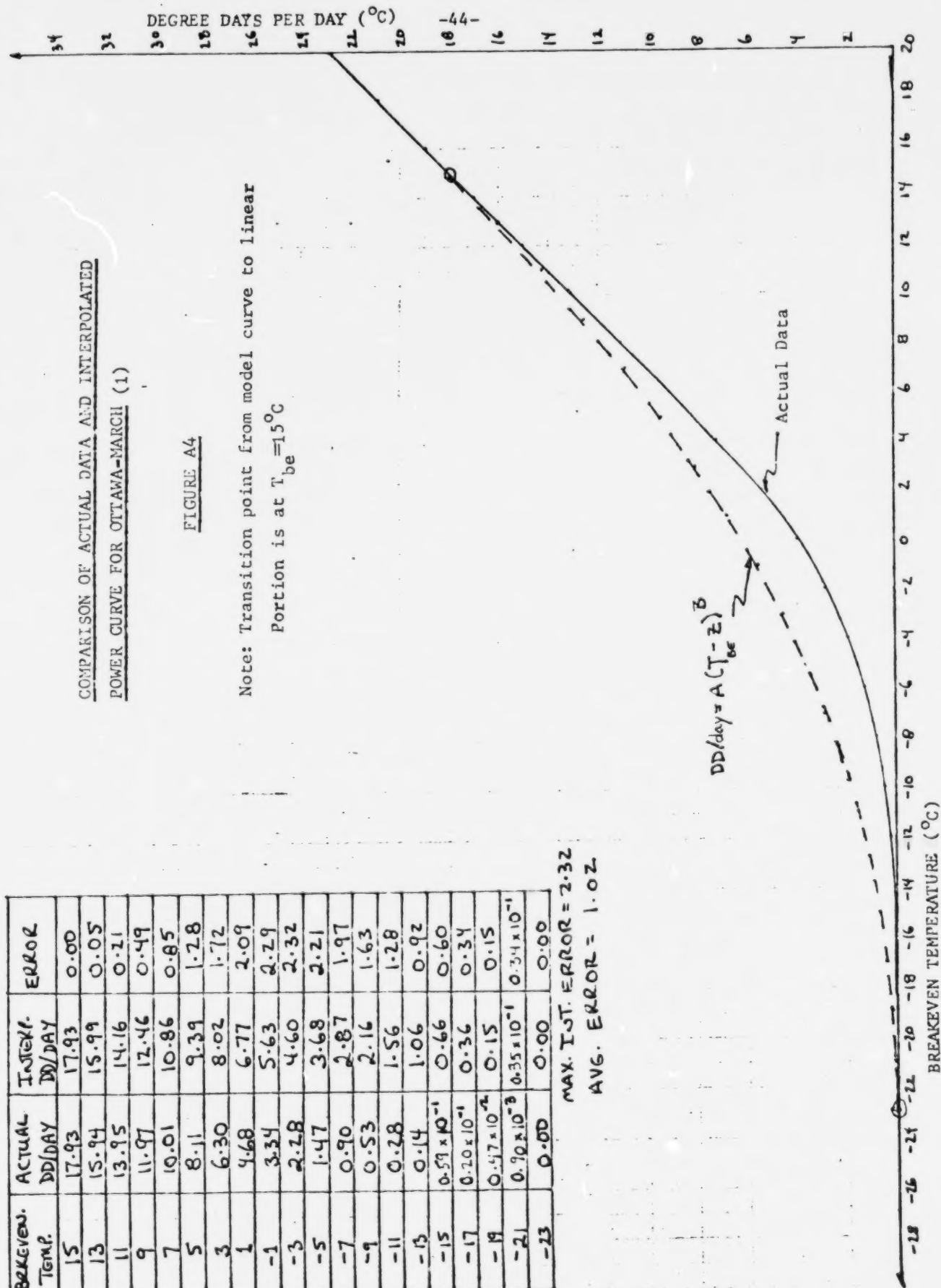
MAX. INT. ERROR = 2.32

AVG. ERROR = 1.02

COMPARISON OF ACTUAL DATA AND INTERPOLATED
POWER CURVE FOR OTTAWA-MARCH (1)

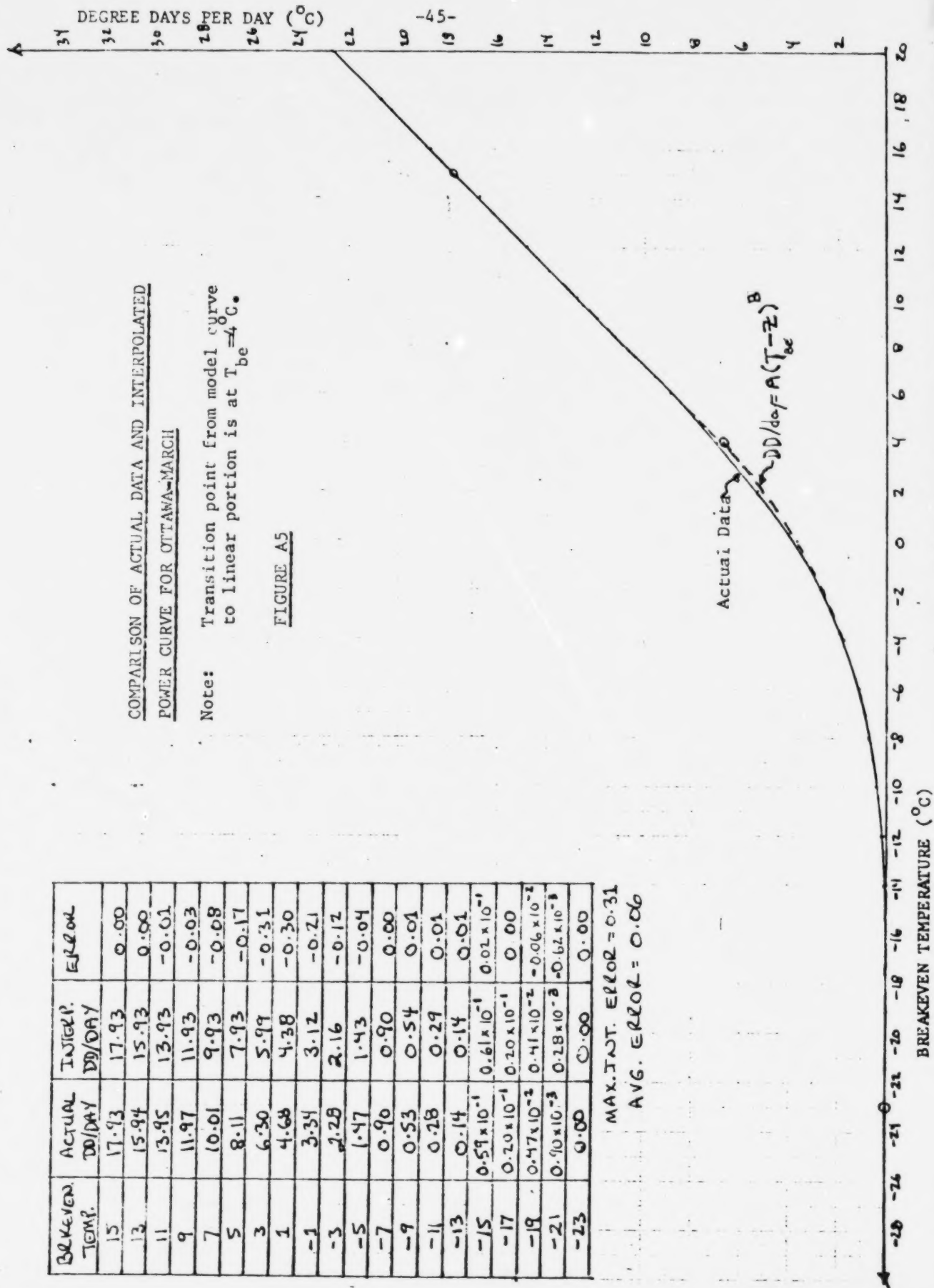
FIGURE A4

Note: Transition point from model curve to linear
Portion is at $T_{be} = 15^{\circ}\text{C}$



BREAKEVEN TEMP.	ACTUAL DD/DAY	INTERP. DD/DAY	ERROR
15	17.93	17.93	0.00
13	15.94	15.93	0.00
11	13.95	13.93	-0.01
9	11.97	11.93	-0.03
7	10.01	9.93	-0.08
5	8.11	7.93	-0.17
3	6.30	5.99	-0.31
1	4.68	4.38	-0.30
-1	3.34	3.12	-0.21
-3	2.28	2.16	-0.12
-5	1.47	1.43	-0.04
-7	0.90	0.90	0.00
-9	0.53	0.54	0.01
-11	0.28	0.29	0.01
-13	0.14	0.14	0.01
-15	0.59×10^{-1}	0.61×10^{-1}	0.02×10^{-1}
-17	0.20×10^{-1}	0.20×10^{-1}	0.00
-19	0.47×10^{-2}	0.41×10^{-2}	-0.06×10^{-2}
-21	0.90×10^{-3}	0.28×10^{-2}	-0.62×10^{-3}
-23	0.00	0.00	0.00

MAX. INST. ERROR = 0.31
AVG. ERROR = 0.06



A.5 FITTED POLYNOMIAL OF DEGREE TWO

The concept of a fitted curve was discussed briefly in Section 4.3. Generally, a fitted curve is one in which the error between the fitted curve and the actual data is minimized. Residuals, (the difference between the fitted curve and the actual data), must therefore be minimized. Normally a so called least squares approach is used. With this method, the total sum of all residuals, squared, is minimized. This takes care of the opposite contribution of positive and negative residuals. Mathematically; fitting a function (DD/day) with this method, involves minimizing:

$$R = \sum_{i=1}^n ((DD/day)_i - \overline{(DD/day)})^2 \quad [A12]$$

Where:

R = residual $(^{\circ}C)^2$

n = number of data points

$(DD/day)_i$ = degree days per day at breakeven temperature T_i

and:

$(DD/day)_i$ = model equation

Turning our consideration to the fitting of a second order polynomial, equation [A7] becomes:

$$R = \sum_{i=1}^n [(DD/day)_i - (A + BT_i + CT_i^2)]^2 \quad [A13]$$

To find the constants A,B&C, the derivative of the above equation is taken with respect to each constant to yield the so called normal equations³:

$$\begin{aligned} nA + B \sum_{i=1}^n T_i + C \sum_{i=1}^n T_i^2 &= \sum_{i=1}^n (DD/day)_i \\ A \sum_{i=1}^n T_i + B \sum_{i=1}^n T_i^2 + C \sum_{i=1}^n T_i^3 &= \sum_{i=1}^n \{T_i (DD/day)_i\} \\ A \sum_{i=1}^n T_i^2 + B \sum_{i=1}^n T_i^3 + C \sum_{i=1}^n T_i^4 &= \sum_{i=1}^n \{T_i^2 (DD/day)_i\} \end{aligned} \quad [A14]$$

Where:

n = number of points fitted.

For comparison purposes, the actual weather the data for Ottawa - March was substituted into equation [A15] to yield the normal equations shown below:

$$\begin{aligned} 20A - 80B + 2980C &= 97.91 \\ -80A + 2980B - 33200C &= 836.81 \\ 2980A - 33200B + 895156C &= 10,364.71 \end{aligned} \quad [A16]$$

The defining equation therefore becomes:

$$DD/day = (1.99494 \text{ E-02 } T_{be}^2) + (6.21419 \text{ E-01 } T_{be}) + 4.40871 (^\circ\text{C}) \quad [A16]$$

A plot of this fitted curve is shown in figure A6. It can be seen from this figure that the maximum interpolation error, and average errors are less than the specified maximums. The method, however has the following drawbacks:

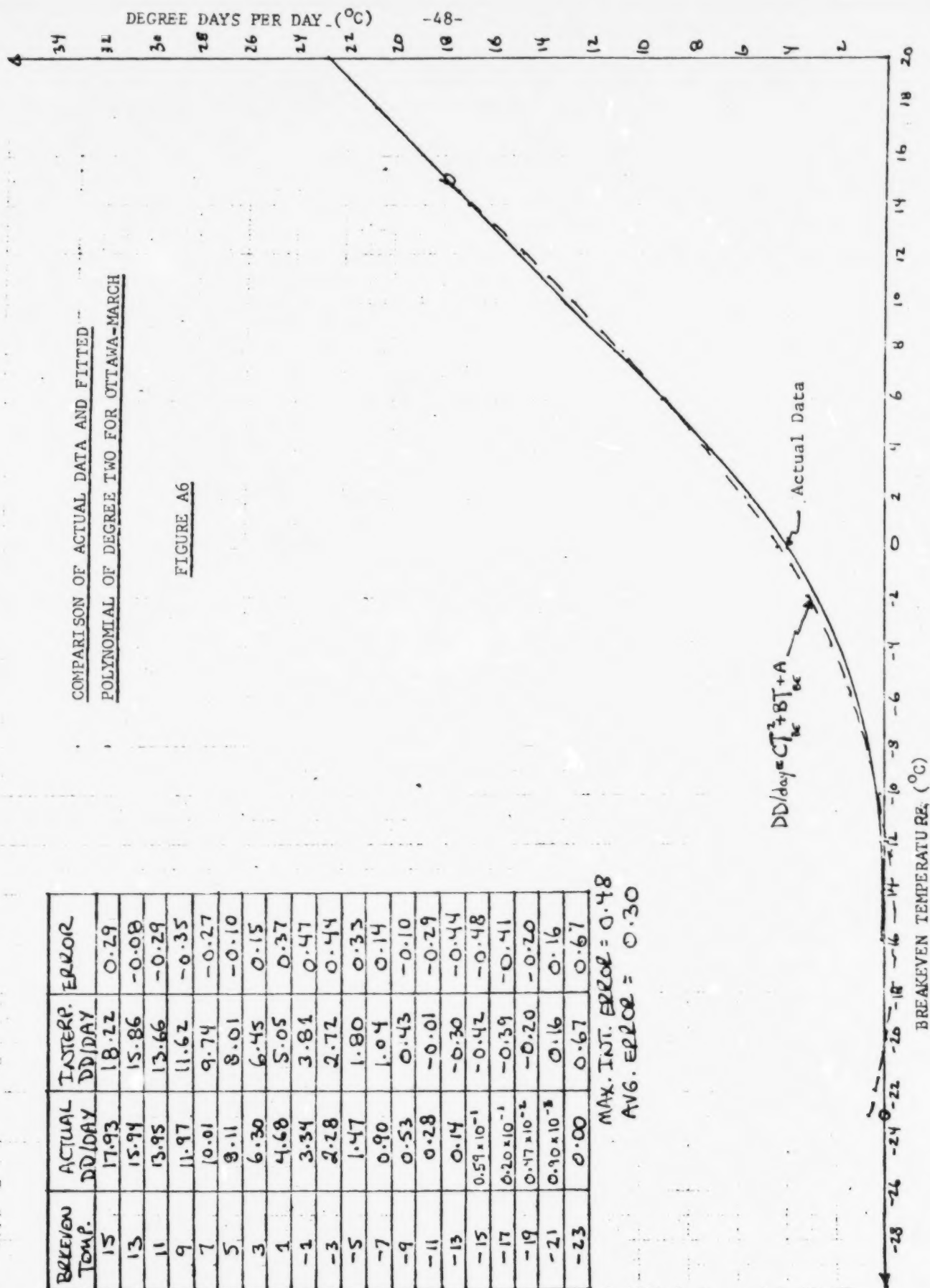
- 1) In a similar fashion to the lagrange polynomial, the equation is unique to the curve for which it is fitted and does not lend itself to transferability amongst similar months.
- 2) The data base would require three real numbers to define each curve.
- 3) There are problems when one tries to define the linear portion of the curve since the transition point to the linear portion of the equation is not obtainable from the information in equation [A16].

BREAKEVEN TEMP.	ACTUAL DD/DAY	INTERP. DD/DAY	ERROR
15	17.93	18.22	0.29
13	15.94	15.86	-0.08
11	13.95	13.66	-0.29
9	11.97	11.62	-0.35
7	10.01	9.74	-0.27
5	8.11	8.01	-0.10
3	6.30	6.45	0.15
1	4.68	5.05	0.37
-1	3.34	3.81	0.47
-3	2.28	2.72	0.44
-5	1.47	1.80	0.33
-7	0.90	1.04	0.14
-9	0.53	0.43	-0.10
-11	0.28	-0.01	-0.29
-13	0.14	-0.30	-0.44
-15	0.59×10^{-1}	-0.42	-0.48
-17	0.20×10^{-1}	-0.39	-0.41
-19	0.47×10^{-2}	-0.20	-0.20
-21	0.90×10^{-3}	0.16	0.16
-23	0.00	0.67	0.67

MAX. INT. ERROR = 0.48
AVG. ERROR = 0.30

COMPARISON OF ACTUAL DATA AND FITTED
POLYNOMIAL OF DEGREE TWO FOR OTTAWA-MARCH

FIGURE A6



A.6 FITTED POWER CURVE

A power curve, when fitted, represents a similar problem to that of fitting a polynomial. In this case the residual to be minimized is:

$$R = \sum_{i=1}^n [(DD/day)_i - (B(T_i - Z)^A)]^2 \quad [A17]$$

here:

$$(DD/day)_i = B(T_i - Z)^A (^\circ C) \quad [A18]$$

Taking the partial derivatives of equation [A17] and setting it equal to zero does not normally lead to a solution. Therefore the natural logarithm of equation [A18] was taken, and the resulting linear equation is fitted. This yields the defining equations below³:

$$A = \frac{n \sum_{i=1}^n [(\ln T_i)(\ln(DD/day)_i)] - [\sum_{i=1}^n \ln T_i] [\sum_{i=1}^n \ln(DD/day)_i]}{[n \sum_{i=1}^n \ln T_i^2] - [\sum_{i=1}^n (\ln T_i)^2]} \quad [A19]$$

$$B = \frac{[\sum_{i=1}^n \ln T_i^2] [\sum_{i=1}^n \ln(DD/day)_i] - [\sum_{i=1}^n \ln T_i \ln(DD/day)_i] [\sum_{i=1}^n \ln T_i]}{n \sum_{i=1}^n \ln T_i^2 - (\sum_{i=1}^n \ln T_i)^2}$$

Where:

n = number of points fitted

For Ottawa - March the constants are:

A = 2.60745

B = 1.75983 E-03

Therefore equation [A18] becomes:

$$DD/day = 1.75983 \text{ E-03 } (T_{be} + 19)^{2.60745} (^\circ C) \quad [A20]$$

NOTES:

- 1) Only data for the range $-19 \leq T_{be} \leq 15$ was fitted since this shows a better fit than if the breakeven range were: $Z \leq T_{be} \leq 15$ (where $Z = -23^\circ C$).

- 2) If this method were chosen, the optimum point of fixation to the x-axis would be found by varying the attachment point until the best fit was found, in a similar fashion to the method used to find the chosen power curve.

A plot of this fitted curve is shown, compared to the actual data in figure A7. It can be seen that not only are the errors too large, but the familiar problem of defining the straight line portion of the graph again surfaces.

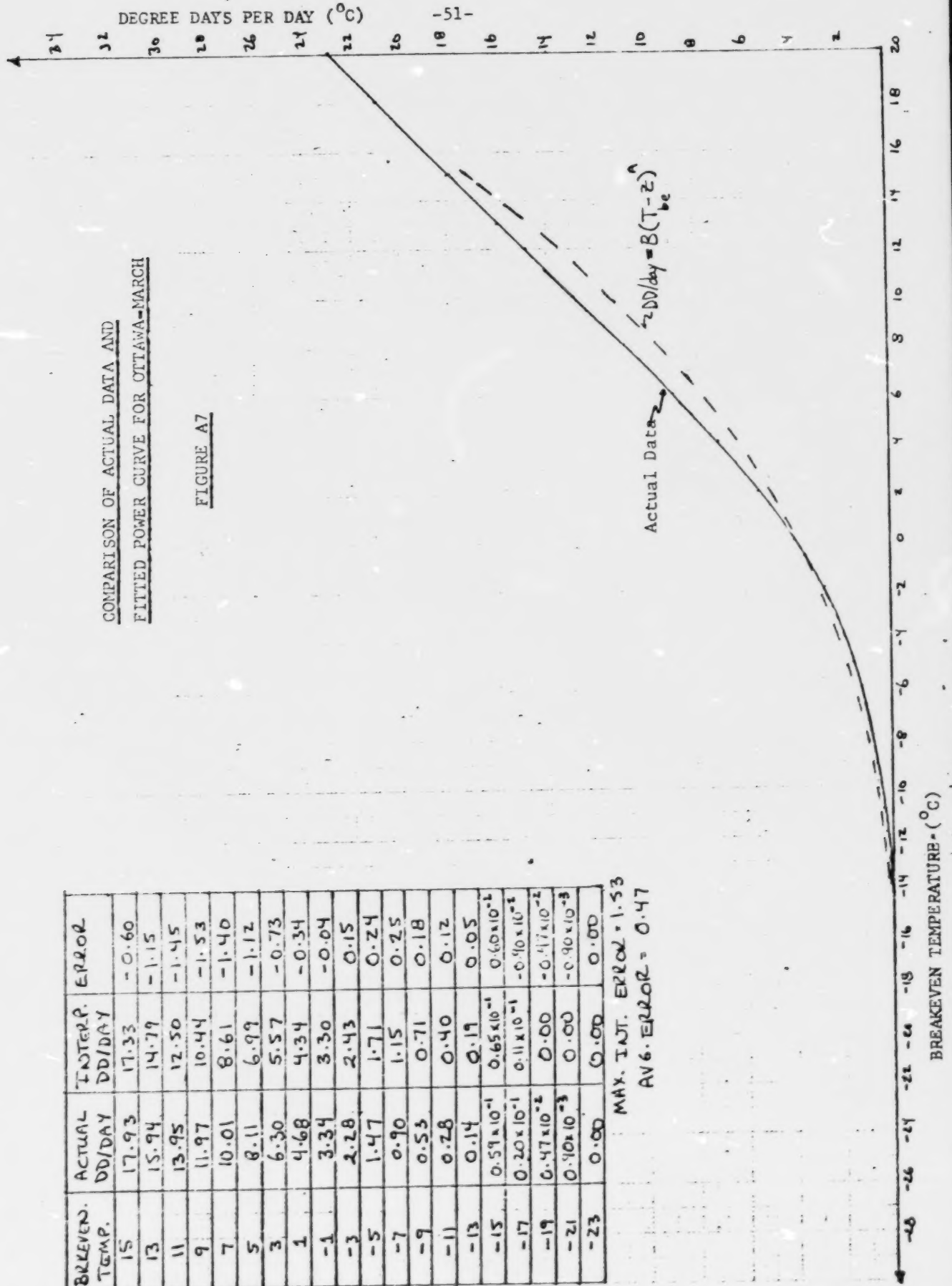
BREKVEN. TEMP.	ACTUAL DD/DAY	INTERP. DD/DAY	ERROR
15	17.93	17.33	-0.60
13	15.94	14.79	-1.15
11	13.95	12.50	-1.45
9	11.97	10.44	-1.53
7	10.01	8.61	-1.40
5	8.11	6.99	-1.12
3	6.30	5.57	-0.73
1	4.68	4.34	-0.34
-1	3.34	3.30	-0.04
-3	2.28	2.43	0.15
-5	1.47	1.71	0.24
-7	0.90	1.15	0.25
-9	0.55	0.71	0.18
-11	0.28	0.40	0.12
-13	0.14	0.19	0.05
-15	0.59×10^{-1}	0.65×10^{-1}	0.60×10^{-2}
-17	0.20×10^{-1}	0.11×10^{-1}	-0.90×10^{-2}
-19	0.47×10^{-2}	0.00	-0.47×10^{-2}
-21	0.90×10^{-3}	0.00	-0.90×10^{-3}
-23	0.00	0.00	0.00

MAX. INT. ERROR = 1.53

AVG. ERROR = 0.47

COMPARISON OF ACTUAL DATA AND
FITTED POWER CURVE FOR OTTAWA-MARCH

FIGURE A7



A.7 FITTED EXPONENTIAL CURVE

Part A (simple version)

The general form of an exponential curve is:

$$DD/day = B(\exp(AT_{be})) \quad (^\circ C) \quad [A21]$$

The clue as to whether or not this curve will fit a set of data, is to plot $\ln[(DD/day)_i]$ versus T_i and see if a straight line is obtained. This has been done in figure A8. As can be seen from the figure, the resulting curve is not linear, thus indicating that to further proceed with the model would be futile.

Part B (advanced version)

A curve of the form:

$$DD/day = B(\exp [\ln(T_{be})]) \quad (^\circ C) \quad [A22]$$

will fit a set of data, if, when $\ln[(DD/day)_i]$ versus $\ln(T_i)$ is plotted, a straight line results. Such a plot is shown in figure A9. It can be seen that this figure contains a fairly straight line, which indicates that the above form of curve may provide a suitable model of the actual data.

Following a similar procedure to that presented in Appendix A.5, the defining equation becomes:

$$DD/day = 1.75983 \text{ E-03} \quad (\exp[2.60745 \ln (T_{be} + 19)]) \quad (^\circ C) \quad [A23]$$

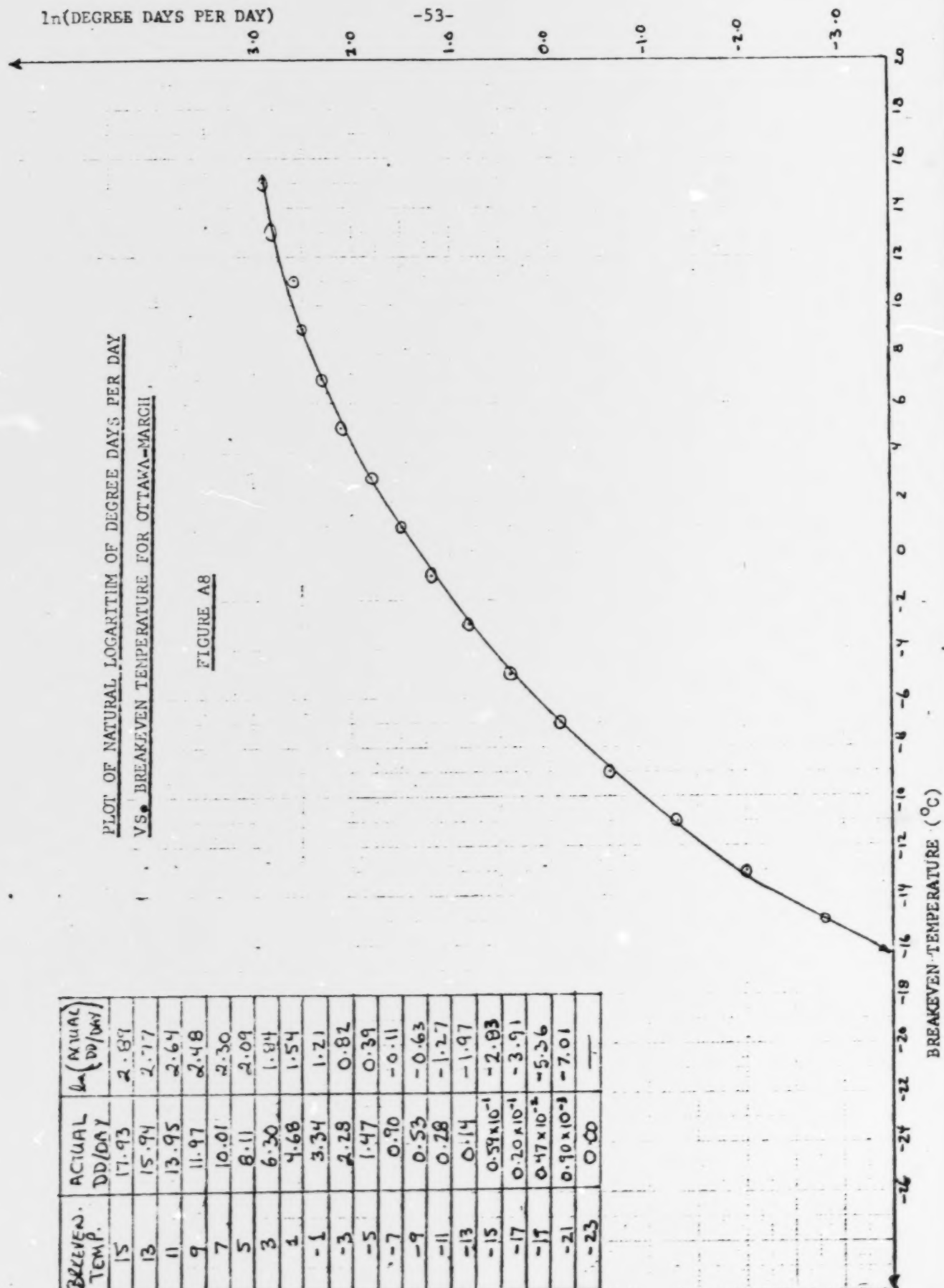
NOTES:

- 1) Only data for the range $-19 \leq T_{be} \leq 15$ $^\circ C$ was used to calculate equation [A23] for the same reasons given in Appendix A.6.

The fitted exponential curve is plotted in figure A10. It can be observed from this figure that the errors are too large. In addition three constants are required to define the curve, (thus there is no advantage over the power curve), and information defining linear portion of the curve, cannot be obtained from equation [A23].

BREAKEVEN TEMP.	ACTUAL DD/DAY	$\ln(\text{ACTUALDD/DAY})$
15	17.93	2.89
13	15.94	2.77
11	13.95	2.64
9	11.97	2.48
7	10.01	2.30
5	8.11	2.09
3	6.30	1.84
1	4.68	1.54
-1	3.34	1.21
-3	2.28	0.82
-5	1.47	0.39
-7	0.90	-0.11
-9	0.53	-0.63
-11	0.28	-1.27
-13	0.14	-1.97
-15	0.59×10^{-1}	-2.83
-17	0.20×10^{-1}	-3.91
-19	0.47×10^{-2}	-5.26
-21	0.90×10^{-3}	-7.01
-23	0.00	—

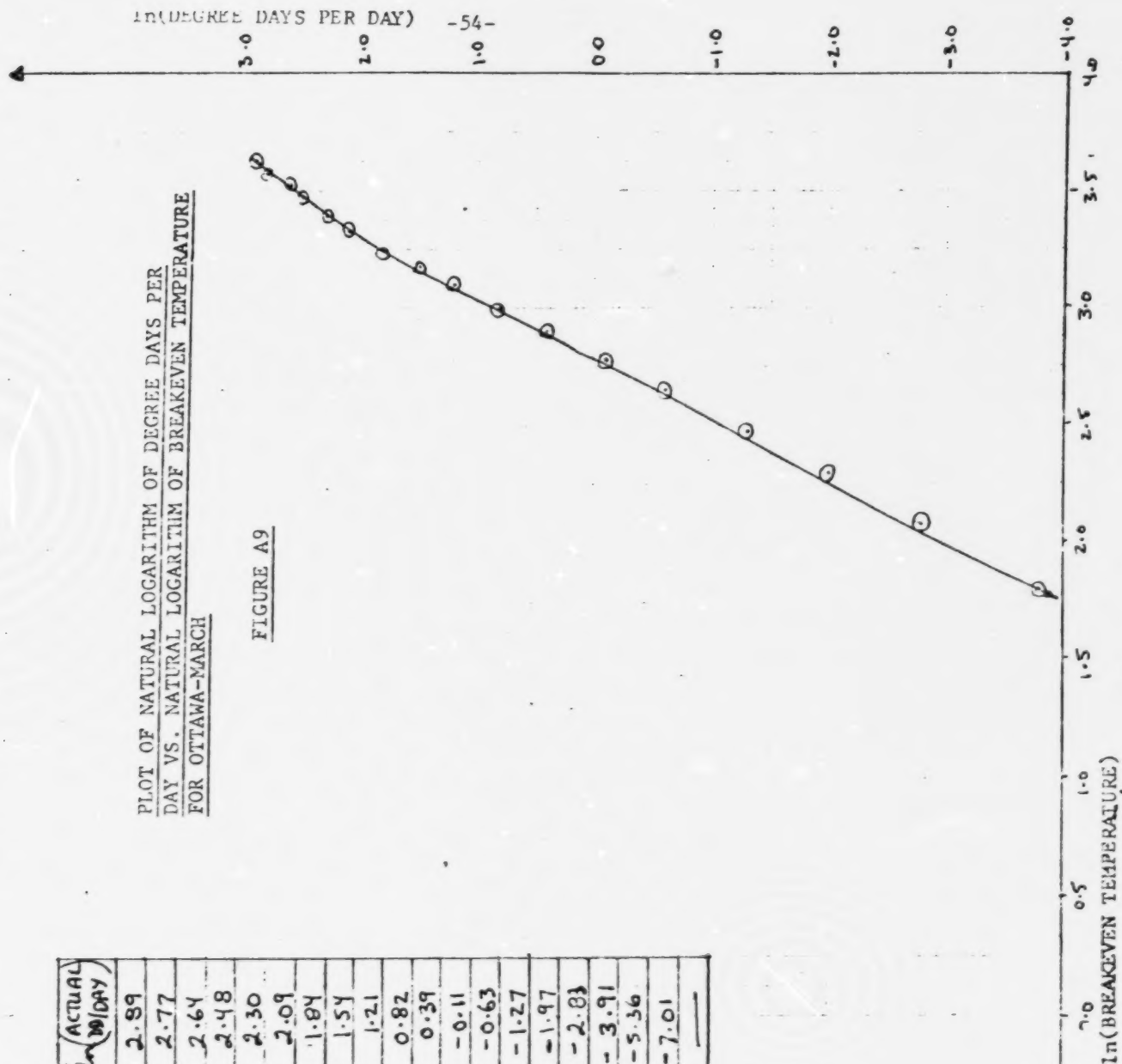
FIGURE A8
 PLOT OF NATURAL LOGARITHM OF DEGREE DAYS PER DAY
 VS. BREAKEVEN TEMPERATURE FOR OTTAWA-MARCH



BREKEN. TEMP. (T)	(T-z)	ACTUAL DD/DAY	$\ln(T-z)$	$\ln(\text{ACTUALDD/DAY})$
15	38	17.93	3.64	2.89
13	36	15.94	3.58	2.77
11	34	13.95	3.53	2.64
9	32	11.97	3.47	2.48
7	30	10.01	3.40	2.30
5	28	8.11	3.33	2.09
3	26	6.30	3.26	1.84
1	24	4.68	3.18	1.54
-1	22	3.34	3.09	1.21
-3	20	2.28	3.00	0.82
-5	18	1.47	2.89	0.39
-7	16	0.90	2.77	-0.11
-9	14	0.53	2.64	-0.63
-11	12	0.28	2.49	-1.27
-13	10	0.14	2.30	-1.97
-15	8	0.54×10^{-1}	2.08	-2.83
-17	6	0.20×10^{-1}	1.79	-3.91
-19	4	0.47×10^{-2}	1.39	-5.36
-21	2	0.40×10^{-3}	0.69	-7.01
-23	0	0.00	—	—

PLOT OF NATURAL LOGARITHM OF DEGREE DAYS PER
 DAY VS. NATURAL LOGARITHM OF BREAKEVEN TEMPERATURE
 FOR OTTAWA-MARCH

FIGURE A9

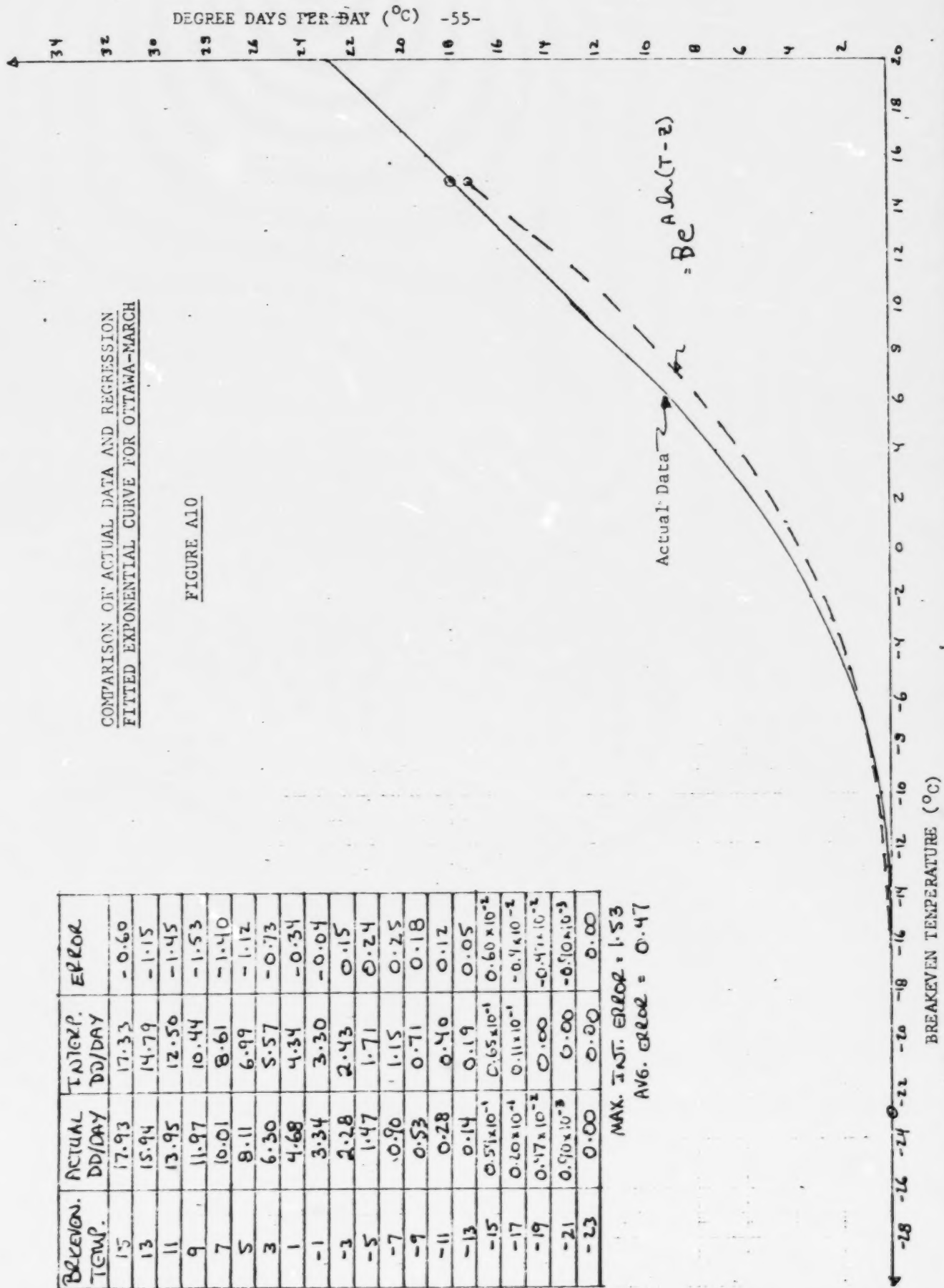


BREAKEVEN TEMP.	ACTUAL DD/DAY	INTERP. DD/DAY	ERROR
15	17.93	17.33	-0.60
13	15.94	14.79	-1.15
11	13.95	12.56	-1.45
9	11.97	10.44	-1.53
7	10.01	8.61	-1.40
5	8.11	6.99	-1.12
3	6.30	5.57	-0.73
1	4.68	4.34	-0.34
-1	3.34	3.30	-0.04
-3	2.28	2.43	0.15
-5	1.47	1.71	0.24
-7	0.90	1.15	0.25
-9	0.53	0.71	0.18
-11	0.28	0.40	0.12
-13	0.14	0.19	0.05
-15	0.51×10^{-1}	0.65×10^{-1}	0.60×10^{-2}
-17	0.20×10^{-1}	0.11×10^{-1}	-0.9×10^{-2}
-19	0.17×10^{-2}	0.00	-0.47×10^{-2}
-21	0.50×10^{-3}	0.00	-0.50×10^{-3}
-23	0.00	0.00	0.00

MAX. INST. ERROR = 1.53
 AVG. ERROR = 0.47

COMPARISON OF ACTUAL DATA AND REGRESSION
 FITTED EXPONENTIAL CURVE FOR OTTAWA-MARCH

FIGURE A10

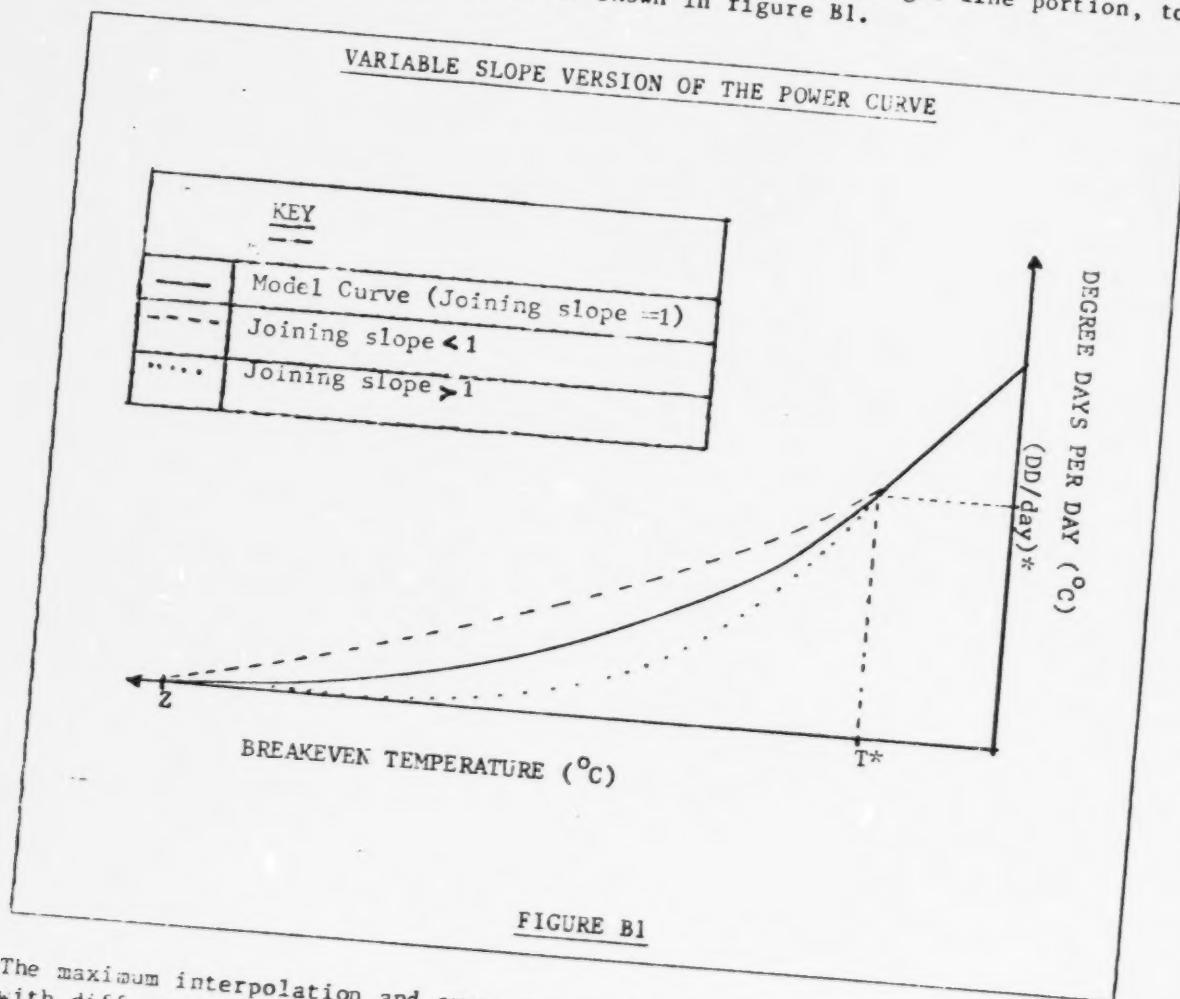


APPENDIX B

VARIABLE SLOPE VERSION
OF THE POWER CURVE

B. VARIABLE SLOPE VERSION OF THE POWER CURVE

The variable slope version of the power curve still requires the power curve to transition to the linear portion of the actual data, however it allows the slope at which the fitted curve transitions to the straight line portion, to vary. This type of power curve is shown in figure B1.



The maximum interpolation and average errors were again computed, for curves with different slopes, at each transition point in order to find the best transition point. A comparison of these errors for the variable slope version and the chosen version are shown in Appendix C.

APPENDIX C

COMPARISON OF CHOSEN POWER
CURVE AND VARIABLE SLOPE VERSION

C. COMPARISON OF CHOSEN POWER CURVE VARIABLE, SLOPE VERSION

This appendix contains a comparison of the maximum interpolation and average errors for both the chosen power curve (where the transition slope was unity) and the variable slope version of the power curve (wherein the transition slope was variable). Table C1 shows the comparison.

CITY	MONTH	MAXIMUM INTERPOLATION ERROR (°C)		AVERAGE ERROR (°C)	
		CHOSEN VERSION	VARIABLE SLOPE VERSION	CHOSEN VERSION	VARIABLE SLOPE VERSION
OW.	JAN.	0.34	= 0.34	0.10	= 0.10
OW.	FEB.	0.29	= 0.29	0.08	= 0.03
OW.	MAR.	0.32	= 0.27	0.06	= 0.06
OW.	APR.	0.56	= 0.56	0.19	= 0.19
OW.	MAY	0.40	= 0.40	0.17	= 0.17
OW.	JUNE	0.21	= 0.21	0.07	= 0.07
OW.	JULY	0.14	= 0.14	0.04	= 0.04
OW.	AUG.	0.20	= 0.20	0.05	= 0.05
OW.	SEPT.	0.36	= 0.36	0.12	= 0.12
OW.	OCT.	0.38	= 0.38	0.14	= 0.14
OW.	NOV.	0.41	= 0.48	0.11	= 0.11
OW.	DEC.	0.29	= 0.29	0.08	= 0.08
VAN.	JAN.	0.14	= 0.14	0.03	= 0.03
VAN.	FEB.	0.14	= 0.20	0.04	= 0.03
VAN.	MAR.	0.14	= 0.18	0.06	= 0.04
VAN.	APR.	0.26	= 0.26	0.05	= 0.05
VAN.	MAY	0.24	= 0.29	0.10	= 0.10
VAN.	JUNE	0.33	= 0.29	0.11	= 0.11
VAN.	JULY	0.25	= 0.25	0.10	= 0.10
VAN.	AUG.	0.28	= 0.28	0.10	= 0.10
VAN.	SEPT.	0.26	= 0.26	0.06	= 0.06
VAN.	OCT.	0.14	= 0.18	0.06	= 0.04
VAN.	NOV.	0.15	= 0.15	0.02	= 0.02
VAN.	DEC.	0.16	= 0.15	0.04	= 0.03
SAS.	JAN.	0.44	= 0.44	0.13	= 0.13
SAS.	FEB.	0.29	= 0.29	0.11	= 0.11
SAS.	MAR.	0.20	= 0.20	0.05	= 0.05
SAS.	APR.	0.48	= 0.48	0.18	= 0.18
SAS.	MAY	0.40	= 0.40	0.19	= 0.19
SAS.	JUNE	0.53	= 0.43	0.14	= 0.13
SAS.	JULY	0.15	= 0.25	0.08	= 0.07
SAS.	AUG.	0.47	= 0.37	0.11	= 0.11
SAS.	SEPT.	0.44	= 0.44	0.19	= 0.19
SAS.	OCT.	0.63	= 0.63	0.17	= 0.17
SAS.	NOV.	0.23	= 0.23	0.04	= 0.04
SAS.	DEC.	0.10	= 0.10	0.03	= 0.03

TABLE C1

COMPARISON OF CHOSEN POWER CURVE AND VARIABLE SLOPE VERSION

NOTE: The equalities have been noted for ease of comparison.

Although it can be seen that the variable slope version of the power curve produces slightly smaller average errors for some months, the "best" intercepting slope must also be contained in the data base. This is shown from the curve's defining equation:

$$DD/day = A (T_{be} - Z)^B \text{ (}^\circ\text{C)} \quad [8]$$

Now the derivative represents the best slope M.

$$M = BA (T_{be} - Z)^{B-1}$$

As before:

$$T^* = Z + \exp [\ln (M/BA)/(B-1)] \text{ (}^\circ\text{C)} \quad [11]$$

$$(DD/day)^* = A (T^* - Z)^B \text{ (}^\circ\text{C)} \quad [12]$$

For the linear portion:

$$DD/day = (T_{be} - T^*) + (DD/day)^* \quad [9]$$

Considering the above equations in reverse order; the linear portion of the model, (equation [9]), can not be defined without first finding T^* , but the equation for T^* includes the best slope M.

Therefore, the positive effect of slightly lower average errors is outweighed by the need to specify four defining constants. (A, B, Z and M), rather than three for the chosen version, i.e. (A, B and Z).

APPENDIX D

DEFINING CONSTANTS OF POWER CURVES
FOR OTTAWA, VANCOUVER AND SASKATOON

CITY	MONTH	A	B	Z
OTT.	JAN.	1.000763E-04	3.258126E+00	-35
OTT.	FEB.	5.350061E-05	3.458188E+00	-33
OTT.	MAR.	1.849083E-05	3.894182E+00	-23
OTT.	APR.	1.025482E-04	3.439986E+00	-13
OTT.	MAY	1.691426E-04	3.275117E+00	- 6
OTT.	JUNE	1.373104E-04	3.442098E+00	2
OTT.	JULY	1.825599E-04	3.498320E+00	7
OTT.	AUG.	4.228087E-05	3.824231E+00	3
OTT.	SEPT.	2.223527E-04	3.273523E+00	- 2
OTT.	OCT.	6.660138E-04	2.984033E+00	- 7
OTT.	NOV.	1.075211E-05	4.047005E+00	-19
OTT.	DEC.	1.535059E-04	3.217046E+00	-29
VAN.	JAN.	1.660417E-06	4.656604E+00	-17
VAN.	FEB.	3.104549E-05	4.102045E+00	- 9
VAN.	MAR.	1.371552E-05	4.364123E+00	- 8
VAN.	APR.	6.775397E-05	4.102623E+00	- 2
VAN.	MAY	1.059270E-03	3.110517E+00	2
VAN.	JUNE	9.881354E-05	3.902683E+00	4
VAN.	JULY	8.833194E-04	3.170537E+00	7
VAN.	AUG.	2.190155E-04	3.635016E+00	6
VAN.	SEPT.	1.383898E-05	4.424018E+00	1
VAN.	OCT.	1.233259E-05	4.398203E+00	- 4
VAN.	NOV.	5.780718E-06	4.641711E+00	- 8
VAN.	DEC.	1.179897E-07	5.332395E+00	-18
SASK.	JAN.	2.887112E-04	2.943710E+00	-43
SASK.	FEB.	3.297364E-04	2.923205E+00	-39
SASK.	MAR.	2.053135E-05	3.564269E+00	-37
SASK.	APR.	1.779129E-04	3.261082E+00	-15
SASK.	MAY	5.210487E-04	2.944570E+00	- 7
SASK.	JUNE	3.091573E-05	3.779152E+00	- 3
SASK.	JULY	9.540037E-04	2.904992E+00	5
SASK.	AUG.	3.127943E-04	3.202531E+00	2
SASK.	SEPT.	2.974268E-04	3.097623E+00	- 8
SASK.	OCT.	1.312194E-06	4.393133E+00	-22
SASK.	NOV.	5.727606E-05	3.402862E+00	-31
SASK.	DEC.	7.025664E-04	2.744430E+00	-36

DEFINING CONSTANTS OF POWER CURVES FOR
OTTAWA, VANCOUVER AND SASKATOON

TABLE D1

APPENDIX E

INVESTIGATION OF POSSIBLE REDUCTIONS
OF THE REQUIRED DATA BASE SIZE (STUDY AND RESULTS)

<u>APPENDIX</u>	<u>CONTENTS</u>	<u>PAGE</u>
E.1	Least number of significant figures required to define the A&B constants for a given month	
E.1.1	Outline of tests made	
E.1.2	Accuracy criteria	
E.1.3	Results of tests	
E.2	Transferability of a month's A&B constants to other similar months	
E.2.1	Outline of tests made	
E.2.2	Accuracy criteria	
E.2.3	Results of tests	

E. INVESTIGATION OF POSSIBLE SAVINGS IN THE REQUIRED
DATA BASE SIZE (SUMMARY OF STUDY AND RESULTS)

E.1 LEAST NUMBER OF SIGNIFICANT FIGURES REQUIRED TO DEFINE THE A&B CONSTANTS
FOR A GIVEN MONTH

The Carleton University Fortran compiler uses a seven decimal place variable when it is declared "real" in a computer program. Therefore the A&B constants, computed for each month, are accurate to seven decimal places. Less accuracy may be used, however, to define a given months degree day per day versus breakeven temperature model.

E.1.1 OUTLINE OF TESTS MADE

Since the A&B constants used to model the weather data for each month are accurate to seven decimal places, yielding the accuracy noted in Table 1, savings in data base size may be possible by using fewer significant figures when defining these two constants.

Tests were performed to determine the minimum number of significant figures in both A&B, required to meet the accuracy specifications stated below. In the tests made the number of significant figures was allowed to increase from the number required to first meet the accuracy specifications, up to seven. This was to see how much, if any, accuracy was gained by using the more accurate defining constants for the power curve.

E.1.2 ACCURACY CRITERIA

For the tests outlined above, the upper limits on the maximum interpolation error and the average error were as follows:

Maximum interpolation error	0.7 °C
and	
Average error	0.6 °C

E.1.3 RESULTS OF TESTS

A computer aided study was carried out to find the least number of significant figures required to define the A&B constants for a given month. This study has been summarized in Tables E1, E2, E3 for Ottawa, Vancouver and Saskatoon respectively. These tables list the maximum interpolation error, average error and average total error between the actual data and the modelling curve for each month, as the number of significant figures in the A&B constants was varied from one to seven.

Table E4 was compiled from Tables E1, E2, E3 and shows how the number of significant figures used to define A&B, affects both the largest maximum interpolation error and the largest average error for the three cities considered in this report.

SUMMARY OF RESULTS FROM STUDY OF; THE LEAST NUMBER OF SIGNIFICANT FIGURES REQUIRED TO DEFINE THE A&B CONSTANTS FOR A GIVEN MONTH: OTTAWA																											
MONTH	MAXIMUM INTERPOLATION ERROR (°C)							AVERAGE ERROR (°C)							AVERAGE TOTAL ERROR (°C)												
	NUMBER OF SIGNIFICANT FIGURES IN A&B CONSTANTS							NUMBER OF SIGNIFICANT FIGURES IN A&B CONSTANTS							NUMBER OF SIGNIFICANT FIGURES IN A&B CONSTANTS												
	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7						
JAN.	*	*	0.29	0.34	0.34	0.34	0.34	-	-	0.11	0.11	0.10	0.10	0.10	-	-	0.04	0.00	0.01	0.01	0.01	0.01					
FEB.	*	*	0.25	0.30	0.29	0.29	0.29	-	-	0.09	0.08	0.08	0.08	0.08	-	-	0.02	0.01	0.01	0.01	0.01	0.01					
MAR.	*	0.36	0.39	0.32	0.32	0.32	0.32	-	0.09	0.11	0.06	0.07	0.06	0.07	-	-0.09	-0.10	-0.06	-0.04	-0.06	-0.06						
APR.	*	*	0.53	0.56	0.56	0.56	0.56	-	-	0.17	0.19	0.18	0.18	0.19	-	-	-0.06	-0.08	-0.07	-0.07	-0.08						
MAY	*	0.35	0.30	0.40	0.40	0.39	0.39	-	0.17	0.16	0.17	0.17	0.17	0.17	-	0.17	0.04	0.02	0.02	0.02	0.02						
JUNE	*	0.49	0.23	0.21	0.21	0.21	0.21	-	0.10	0.07	0.07	0.07	0.07	0.07	-	-0.04	0.02	0.03	0.03	0.03	0.03						
JULY	*	0.16	0.13	0.14	0.14	0.14	0.14	-	0.04	0.04	0.04	0.04	0.04	0.04	-	0.00	0.01	0.01	0.01	0.01	0.01						
AUG.	*	0.35	0.22	0.22	0.20	0.20	0.20	-	0.07	0.05	0.05	0.05	0.05	0.05	-	-0.03	0.00	0.01	0.01	0.01	0.01						
SEPT.	*	0.20	0.43	0.35	0.36	0.36	0.36	-	0.08	0.13	0.12	0.12	0.12	0.12	-	0.08	-0.03	-0.01	-0.02	-0.02	-0.02						
OCT.	*	0.38	0.45	0.38	0.38	0.38	0.38	-	0.13	0.17	0.14	0.14	0.14	0.14	-	0.11	-0.09	-0.05	-0.05	-0.05	-0.05						
NOV.	*	*	0.34	0.41	0.41	0.41	0.41	-	-	0.12	0.11	0.11	0.11	0.11	-	-	0.03	-0.03	-0.02	-0.01	-0.02						
DEC.	*	*	0.20	0.29	0.29	0.29	0.29	-	-	0.11	0.08	0.08	0.09	0.08	-	-	0.06	-0.01	-0.01	0.00	-0.01						

TABLE E1

NOTES: * Denotes "maximum interpolation error was = 0.7°C".
 - Denotes "values not considered relevant since maximum interpolation error was = 0.7°C".

SUMMARY OF RESULTS FROM STUDY OF; THE LEAST NUMBER OF SIGNIFICANT FIGURES REQUIRED TO DEFINE THE A&B CONSTANTS FOR A GIVEN MONTH: VANCOUVER																						
MONTH	MAXIMUM INTERPOLATION ERROR (°C)							AVERAGE ERROR (°C)							AVERAGE TOTAL ERROR (°C)							
	NUMBER OF SIGNIFICANT FIGURES IN A&B CONSTANTS							NUMBER OF SIGNIFICANT FIGURES IN A&B CONSTANTS							NUMBER OF SIGNIFICANT FIGURES IN A&B CONSTANTS							
	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	
JAN.	*	*	0.12	0.14	0.14	0.14	0.14	-	-	0.06	0.05	0.03	0.03	0.03	-	-	0.05	0.03	0.00	0.00	0.00	
FEB.	*	0.15	0.15	0.14	0.14	0.14	0.14	-	0.06	0.06	0.07	0.04	0.04	0.04	-	0.00	0.00	0.05	0.02	0.02	0.02	
MAR.	*	0.51	0.19	0.14	0.14	0.14	0.14	-	0.26	0.07	0.04	0.04	0.04	0.06	-	0.26	-0.02	0.01	0.01	0.01	0.05	
APR.	*	0.27	0.27	0.25	0.26	0.26	0.26	-	0.06	0.06	0.07	0.05	0.05	0.05	-	-0.05	-0.05	0.00	-0.04	-0.04	-0.04	
MAY	*	0.20	0.25	0.24	0.24	0.24	0.24	-	0.09	0.10	0.09	0.10	0.09	0.10	-	0.02	-0.01	0.00	-0.01	0.00	-0.01	
JUNE	*	0.35	0.35	0.33	0.33	0.33	0.33	-	0.11	0.12	0.10	0.10	0.11	0.11	-	-0.07	-0.07	-0.05	-0.05	-0.06	-0.06	
JULY	*	0.18	0.25	0.24	0.25	0.25	0.25	-	0.08	0.10	0.10	0.10	0.10	0.10	-	0.07	0.01	0.02	0.01	0.01	0.01	
AUG.	*	0.55	0.24	0.28	0.28	0.28	0.28	-	0.15	0.10	0.10	0.10	0.10	0.10	-	-0.07	0.00	-0.01	-0.01	-0.01	-0.01	
SEPT.	*	0.42	0.30	0.26	0.26	0.26	0.26	-	0.12	0.09	0.06	0.07	0.06	0.06	-	-0.09	-0.04	-0.01	-0.03	-0.01	-0.01	
OCT.	*	0.21	0.13	0.14	0.14	0.14	0.14	-	0.08	0.06	0.05	0.06	0.05	0.06	-	-0.03	0.04	0.01	0.04	0.01	0.04	
NOV.	*	0.54	0.16	0.14	0.15	0.15	0.15	-	0.23	0.03	0.05	0.02	0.02	0.02	-	-0.23	-0.03	0.02	-0.02	-0.02	-0.02	
DEC.	*	0.51	0.15	0.16	0.16	0.16	0.16	-	0.19	0.05	0.04	0.06	0.04	0.04	-	-0.16	0.00	0.01	0.04	0.01	0.01	

TABLE E2

NOTES: * Denotes "maximum interpolation error was = 0.7°C".
- Denotes "values not considered relevant since maximum interpolation error was = 0.7°C".

SUMMARY OF RESULTS FROM STUDY OF; THE LEAST NUMBER OF SIGNIFICANT FIGURES REQUIRED TO DEFINE THE A&B CONSTANTS FOR A GIVEN MONTH: SASKATOON																						
MONTH	MAXIMUM INTERPOLATION ERROR (°C)							AVERAGE ERROR (°C)							AVERAGE TOTAL ERROR (°C)							
	NUMBER OF SIGNIFICANT FIGURES IN A&B CONSTANTS							NUMBER OF SIGNIFICANT FIGURES IN A&B CONSTANTS							NUMBER OF SIGNIFICANT FIGURES IN A&B CONSTANTS							
	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	
JAN.	*	*	0.56	0.43	0.44	0.44	0.44	-	-	0.20	0.12	0.13	0.13	0.13	-	-	-0.11	-0.02	-0.02	-0.02	-0.02	
FEB.	*	*	0.40	0.30	0.29	0.29	0.29	-	-	0.18	0.12	0.11	0.12	0.11	-	-	-0.08	-0.01	0.00	-0.01	0.00	
MAR.	*	*	0.38	0.22	0.20	0.20	0.20	-	-	0.13	0.05	0.05	0.05	0.05	-	-	-0.11	-0.02	-0.01	-0.01	-0.02	
APR.	*	*	0.50	0.48	0.48	0.48	0.48	-	-	0.19	0.18	0.17	0.18	0.18	-	-	-0.05	-0.04	-0.03	-0.04	-0.04	
MAY	*	*	0.51	0.39	0.40	0.40	0.40	-	-	0.20	0.18	0.19	0.19	0.19	-	-	-0.01	0.03	0.02	0.02	0.02	
JUNE	*	0.22	0.52	0.53	0.52	0.53	0.53	-	0.10	0.13	0.14	0.14	0.14	0.14	-	0.02	-0.04	-0.05	-0.05	-0.05	-0.05	
JULY	*	0.17	0.16	0.15	0.15	0.15	0.15	-	0.08	0.08	0.08	0.08	0.08	0.08	-	0.04	0.04	0.05	0.05	0.05	0.05	
AUG.	*	0.52	0.49	0.46	0.47	0.47	0.47	-	0.12	0.12	0.11	0.11	0.11	0.11	-	-0.06	-0.05	-0.05	-0.05	-0.05	-0.05	
SEPT.	*	0.33	0.40	0.43	0.44	0.44	0.44	-	0.17	0.18	0.19	0.19	0.19	0.19	-	0.03	0.01	0.00	-0.01	-0.01	0.00	
OCT.	*	0.54	*	0.64	0.63	0.63	0.63	-	0.15	-	0.19	0.18	0.18	0.17	-	-0.05	-	-0.09	-0.09	-0.09	-0.08	
NOV.	*	0.37	0.32	0.23	0.23	0.23	0.23	-	0.10	0.08	0.05	0.05	0.04	0.04	-	-0.09	-0.07	-0.01	-0.01	-0.02	-0.02	
DEC.	*	*	0.29	0.12	0.10	0.10	0.10	-	-	0.11	0.04	0.03	0.03	0.03	-	-	-0.09	0.00	0.01	0.01	0.01	

TABLE E3

NOTES: * Denotes "maximum interpolation error was = 0.7°C".
 - Denotes "values not considered relevant since maximum interpolation error was = 0.7°C".

NUMBER OF SIGNIFICANT FIGURES IN A&B	LARGEST MAXIMUM INTERPOLATION ERROR (°C)	LARGEST AVERAGE ERROR (°C)
1	0.7	-
2	0.7	-
3	0.7	-
4	0.64	0.19
5	0.63	0.19
6	0.63	0.19
7	0.63	0.19
<u>TABLE E4</u> <u>BEHAVIOUR OF MAXIMUM ERRORS WITH</u> <u>NUMBER OF SIGNIFICANT FIGURES IN A&B</u>		

Note: - Denotes "values not considered relevant since maximum interpolation error was = 0.7°C.

It can be seen from this table that the largest maximum interpolation error is not affected in its second decimal place after five significant figures are used to define the A&B constants. The largest average error is not affected in its second decimal place, when the number of significant figures is above four.

In addition to the above observations, Tables E1, E2, E3 show the following trends:

- 1) The tendency for all three types of error, is a decrease in magnitude, as the number of significant figures in A&B increase.
- 2) The accuracy criteria specified at the beginning of the report, (Section 3.2), are both met when the number of significant figures is four or more. (Saskatoon - October is the reason that at least four significant figures must be specified.)
- 3) Only small decreases in error magnitude, (considering all three types of error), are yielded by using more than four significant figures to define the A&B constants for a given month.

E.2 TRANSFERABILITY OF A MONTH'S A&B CONSTANTS TO OTHER SIMILAR MONTHS

The transferability of one month's defining constants (A&B) to the data for other similar months is considered in this section.

E.2.1 OUTLINE OF TESTS MADE

The A&B constants for month n, were used to model the weather data for month m using the equation:

$$(DD/day)_m = A_n (T_{be} - Z_m)^B \quad (^\circ C)$$

This equation was used to see if the maximum interpolation error and the average error (between the actual data and the defining equation) were within bounds. This was done while the number of significant figures in the A&B constants was varied from one to seven in order to observe the effect of having more significant figures in the A&B constants.

E.2.2 ACCURACY CRITERIA

The maximum interpolation and average errors were, as before, to be less than 0.7°C and 0.6°C respectively.

E.2.3 RESULTS OF TESTS

A computer aided study was performed to check the transferability of the A&B constants among similar months.

The A&B constants for a given month were used to model the weather data for all other 35 months, (for Ottawa, Vancouver and Saskatoon), to see if there is a match, as described in Section E.2.1 while at the same time the number of significant figures in the A&B constants was varied.

A summary of the matches that occurred is contained in tables F1, F2 and F3.

From the study it was found that the accuracy criteria used, provided for a great number of "matches". In other words, of the 36 months modelled in this study, many of the defining equations used similar A&B constants. These results must be viewed as positive since it means that a smaller data base will be possible.

The following specific observations can be made from the output:

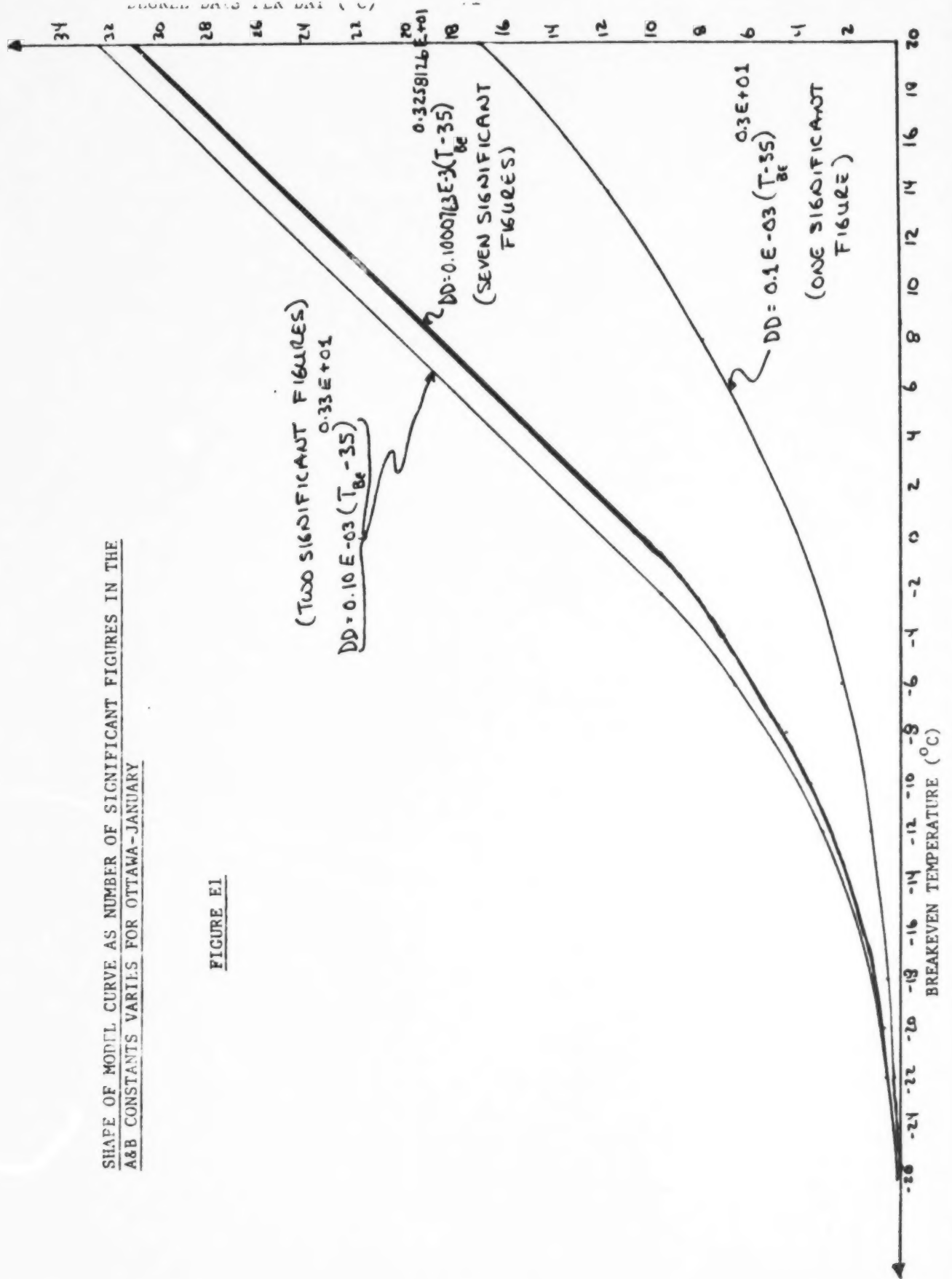
- 1) The matches which occurred, were not necessarily reciprocal. An example will illustrate the meaning of this:

The A&B constants for Ottawa - January provided a good model of the weather data for Saskatoon - November, however the A&B constants for Saskatoon - November did not represent the weather data for Ottawa - January.

- 2) The accuracy with which the A&B constants for any given month modelled the weather data for another month did not always increase as the number of significant figures in A&B increases.
- 3) Generally, there are only small gains in accuracy, (i.e. lower error values), when more significant figures are used to define the A&B constants once a match has been established. (There are also some exceptions where the error values grow larger as the number of significant figures increases.)
- 4) The defining equations for the mild months were less unique than those of the colder months. This can be seen from the fact that the equations for the summer months in Vancouver were transferable to many more months of weather data than were the equations for the cold months in Saskatoon.
- 5) The shape of the curve defined with A&B constants of only one or two significant figures is unlike its shape when a larger number of significant figures is used. This can be seen from Figure E1, for Ottawa - January. This phenomena makes itself evident from the fact that many matches occur with only one or two significant figures in A&B, but the similarity between these months disappears when more accurate constants are used.

SHAPE OF MODEL CURVE AS NUMBER OF SIGNIFICANT FIGURES IN THE
A&B CONSTANTS VARIES FOR OTTAWA-JANUARY

FIGURE E1



APPENDIX F

INVESTIGATION OF POSSIBLE CORRELATION
BETWEEN ANY TWO SIMILAR MONTH'S MEAN MONTHLY TEMPERATURES
(STUDY AND RESULTS.)

F. INVESTIGATION OF POSSIBLE CORRELATION BETWEEN ANY TWO SIMILAR MONTH'S
MEAN MONTHLY TEMPERATURES (STUDY AND RESULTS)

In order to cut down on the amount of future computing needed to discover whether or not the A&B constants used in the defining equation for a given month represent the data of another month, a correlation was sought between the "closeness" of the month's mean monthly temperatures, and whether or not those months were represented by the same A&B constants.

Tables F1, F2 & F3 show the matches defined in Appendix E.2, where for each match, the least number of significant figures used to define A&B, are considered.

The following information is shown in tables F1, F2 & F3 for those months wherein the transferability of a defining equation was established; the mean monthly temperatures for each month (taken from Appendix G), the mean monthly temperature difference, the maximum interpolation error, the average error and the average total error.

From these tables it can be seen that the mean monthly temperature difference is at best, only a crude indicator of the likeliness of a match between the defining equation for one month, and the weather data of another. This can be shown by referring to Appendix G, which contains the mean monthly temperatures for the months considered. The largest spread possible is between Saskatoon - January ($T_{mm} = -17.8^{\circ}\text{C}$) and Ottawa - July ($T_{mm} = 20.7^{\circ}\text{C}$), or 38.5°C total spread. A match, however, occurred once where the mean monthly temperature difference was 34.0°C (i.e. when A&B constants for Saskatoon - December were used to model weather data for Ottawa - July).

NOTE: The mean monthly temperatures were not calculated for the same years from which the hourly data was gathered, and this will have a minor effect on the results presented in Tables F1, F2 and F3.

Summarizing the above results; if the defining constants for a given month also define another months weather data, their mean monthly temperatures may be quite different. This is due to the fact that the curves for any two month's may have the same shape, but have vastly different mean monthly temperatures.

TABLE F1

RELATIONSHIP BETWEEN CLOSENESS OF TWO MONTHS MEAN MONTHLY TEMPERATURES AND WHETHER OR NOT THE DEFINING CONSTANTS FOR ONE REPRESENT THE WEATHER DATA OF THE OTHER.

CITY: OTTAWA

MONTH FOR WHICH A&B CONSTANTS ARE USED TO MODEL SIMILAR MONTHS (MONTH A)	CITY MODELLED (MONTH B)	MONTH MODELLED (MONTH B)	NUMBER OF SIGNIFICANT FIGURES USED TO DEFINE A&B CONSTANTS FOR (MONTH A)	MEAN MONTHLY TEMPERATURE FOR (MONTH A) (°C)	MEAN MONTHLY TEMPERATURE FOR (MONTH B) (°C)	MEAN MONTHLY TEMPERATURE DIFFERENCE (°C)	ERRORS BETWEEN CURVE DEFINED BY A&B CONSTANTS FOR (MONTH A) AND DATA FOR MONTH B		
							MAXIMUM INTERPOLATION ERROR (°C)	AVERAGE ERROR (°C)	AVERAGE TOTAL ERROR (°C)
JAN.	SAS.	FEB.	3	-10.9	-14.4	3.5	0.52	0.14	-0.07
JAN.	SAS.	NOV.	3	-10.9	-6.1	4.8	0.55	0.29	0.29
JAN.	SAS.	DEC.	2	-10.9	-13.3	2.4	0.69	0.25	-0.15
MAR.	OW.	AUG.	1	-3.1	19.3	22.4	0.67	0.13	-0.12
MAR.	OW.	NOV.	2	-3.1	1.4	4.5	0.32	0.14	0.10
MAR.	VAN.	JAN.	2	-3.1	2.8	5.9	0.50	0.25	-0.10
MAR.	SAS.	MAY	1	-3.1	11.7	14.8	0.68	0.18	-0.04
APR.	OW.	MAR.	2	5.6	-3.1	8.7	0.42	0.26	0.26
APR.	OW.	MAY	3	5.6	12.4	6.8	0.22	0.11	0.03
APR.	OW.	AUG.	3	5.6	19.3	13.7	0.59	0.11	-0.05
APR.	SAS.	APR.	3	5.6	3.9	1.7	0.34	0.15	0.05
APR.	SAS.	MAY	3	5.6	11.7	6.1	0.64	0.21	-0.19
APR.	SAS.	JUNE	2	5.6	16.1	10.5	0.59	0.16	0.02
APR.	SAS.	SEPT.	3	5.6	11.1	5.5	0.26	0.10	0.00
MAY	OW.	APR.	2	12.4	5.6	6.8	0.38	0.18	0.13
MAY	OW.	AUG.	2	12.4	19.3	6.9	0.39	0.09	0.01
MAY	SAS.	APR.	2	12.4	3.9	8.5	0.67	0.24	0.24
MAY	SAS.	MAY	2	12.4	11.7	0.7	0.33	0.11	-0.04
MAY	SAS.	JUNE	2	12.4	16.1	3.7	0.55	0.30	0.30
MAY	SAS.	SEPT.	2	12.4	-6.1	18.5	0.61	0.16	0.16
JUNE	OW.	JULY	3	18.2	20.7	2.5	0.65	0.13	-0.13
JUNE	OW.	AUG.	2	18.2	19.3	1.1	0.21	0.08	0.05
JUNE	OW.	SEPT.	3	18.2	14.6	3.6	0.24	0.09	-0.02
JULY	VAN.	FEB.	2	20.7	5.0	15.7	0.19	0.11	0.10
JULY	VAN.	MAR.	2	20.7	6.1	14.6	0.55	0.25	0.25
JULY	VAN.	SEPT.	2	20.7	14.4	6.3	0.52	0.20	0.01
JULY	VAN.	OCT.	2	20.7	10.6	10.1	0.56	0.26	0.26
JULY	VAN.	NOV.	2	20.7	6.1	14.6	0.61	0.36	0.36
JULY	SAS.	JULY	2	20.7	19.4	1.3	0.26	0.07	-0.07
AUG.	OW.	JUNE	2	19.3	18.2	1.1	0.61	0.13	-0.12
AUG.	OW.	JULY	1	19.3	20.7	1.4	0.44	0.10	-0.10
AUG.	OW.	SEPT.	2	19.3	14.6	4.7	0.65	0.21	-0.21
AUG.	VAN.	FEB.	1	19.3	5.0	14.3	0.28	0.13	-0.07
AUG.	VAN.	MAR.	1	19.3	6.1	13.2	0.26	0.09	0.08
AUG.	VAN.	OCT.	1	19.3	10.6	8.7	0.27	0.10	0.10
AUG.	VAN.	NOV.	1	19.3	6.1	13.2	0.32	0.20	0.20
AUG.	SAS.	JULY	1	19.3	19.4	0.1	0.59	0.20	-0.20
AUG.	SAS.	AUG.	1	19.3	17.8	1.5	0.46	0.05	0.04
SEPT.	OW.	JUNE	2	14.6	18.2	3.6	0.21	0.10	0.10
SEPT.	OW.	JULY	2	14.6	20.7	6.1	0.54	0.10	-0.09
SEPT.	OW.	AUG.	2	14.6	19.3	4.7	0.32	0.16	0.16
SEPT.	SAS.	AUG.	2	14.6	17.8	3.2	0.69	0.15	-0.12
OCT.	OW.	JUNE	3	8.7	18.2	9.5	0.61	0.32	0.32
OCT.	OW.	JULY	1	8.7	20.7	12.0	0.17	0.08	0.07
OCT.	VAN.	MAR.	1	8.7	6.1	2.6	0.66	0.40	0.07
OCT.	VAN.	OCT.	1	8.7	10.6	1.9	0.67	0.36	0.00
OCT.	VAN.	NOV.	1	8.7	6.1	2.6	0.67	0.33	0.22
OCT.	SAS.	JULY	1	8.7	19.4	10.7	0.25	0.07	-0.15
OCT.	SAS.	AUG.	1	8.7	17.8	9.1	0.37	0.22	-0.17
NOV.	OW.	MAR.	3	1.4	-3.1	11.8	0.50	0.15	-0.15
NOV.	VAN.	JAN.	3	1.4	2.8	5.9	0.60	0.26	-0.17

NOTE: The A&B constants for Ottawa - February and Ottawa - December did not represent (or model) any of the other 35 months of weather data.

TABLE F2

RELATIONSHIP BETWEEN CLOSENESS OF TWO MONTHS MEAN MONTHLY TEMPERATURES AND WHETHER OR NOT THE DEFINING CONSTANTS FOR ONE REPRESENT THE WEATHER DATA OF THE OTHER.

CITY: VANCOUVER

MONTH FOR WHICH AAB CONSTANTS ARE USED TO MODEL SIMILAR MONTHS (MONTH A)	CITY MODELLED (MONTH B)	MONTH MODELLED (MONTH B)	NUMBER OF SIGNIFICANT FIGURES USED TO DEFINE AAB CONSTANTS FOR (MONTH A)	MEAN MONTHLY TEMPERATURE FOR (MONTH A) (°C)	MEAN MONTHLY TEMPERATURE FOR (MONTH B) (°C)	MEAN MONTHLY TEMPERATURE DIFFERENCE (°C)	ERRORS BETWEEN CURVE DEFINED BY AAB CONSTANTS FOR (MONTH A) AND DATA FOR MONTH B		
							MAXIMUM INTERPOLATION ERROR (°C)	AVERAGE ERROR (°C)	AVERAGE TOTAL ERROR (°C)
JAN.	OW.	MAR.	3	2.8	-3.1	5.9	0.55	0.29	0.12
JAN.	OW.	AUG.	1	2.8	19.3	16.5	0.49	0.06	0.06
JAN.	OW.	NOV.	1	2.8	1.4	1.4	0.70	0.24	0.22
JAN.	VAN.	MAR.	1	2.8	6.1	3.3	0.52	0.19	-0.18
JAN.	VAN.	OCT.	1	2.8	10.6	7.8	0.48	0.15	-0.15
JAN.	VAN.	NOV.	2	2.8	6.1	3.3	0.28	0.07	-0.07
JAN.	SAS.	JUNE	1	2.8	16.1	13.3	0.69	0.20	-0.20
JAN.	SAS.	AUG.	1	2.8	17.8	15.0	0.50	0.19	-0.18
FEB.	OW.	JUNE	1	5.0	18.2	13.2	0.07	0.02	0.02
FEB.	OW.	AUG.	1	5.0	19.3	14.3	0.16	0.07	0.07
FEB.	VAN.	SEPT.	2	5.0	14.4	9.4	0.61	0.18	-0.12
FEB.	VAN.	OCT.	2	5.0	10.6	5.6	0.28	0.15	0.15
FEB.	VAN.	NOV.	2	5.0	6.1	1.1	0.47	0.26	0.26
FEB.	SAS.	JULY	2	5.0	19.4	14.4	0.56	0.20	-0.20
FEB.	SAS.	AUG.	1	5.0	17.8	12.8	0.61	0.20	-0.20
MAR.	OW.	JULY	2	6.1	20.7	14.6	0.47	0.12	-0.12
MAR.	VAN.	FEB.	2	6.1	5.0	1.1	0.24	0.12	0.12
MAR.	VAN.	SEPT.	2	6.1	14.4	8.3	0.42	0.12	-0.09
MAR.	VAN.	OCT.	2	6.1	10.6	4.5	0.54	0.24	0.24
MAR.	VAN.	NOV.	1	6.1	6.1	0.0	0.19	0.10	0.10
MAR.	SAS.	JULY	2	6.1	11.1	5.0	0.55	0.21	-0.21
MAR.	SAS.	AUG.	3	6.1	17.8	11.7	0.33	0.09	-0.04
APR.	OW.	JULY	1	9.4	20.7	11.3	0.42	0.11	0.1
APR.	VAN.	JUNE	1	9.4	15.6	6.2	0.58	0.19	-0.16
APR.	VAN.	AUG.	2	9.4	17.2	7.8	0.21	0.07	-0.03
MAY	OW.	JULY	1	12.2	20.7	8.5	0.61	0.26	0.26
MAY	VAN.	APR.	2	12.2	9.4	2.8	0.52	0.34	0.34
MAY	VAN.	JULY	2	12.2	17.2	5.0	0.20	0.09	0.06
MAY	VAN.	AUG.	2	12.2	17.2	5.0	0.48	0.28	0.28
JUNE	OW.	JULY	2	15.6	20.7	5.1	0.60	0.16	0.16
JUNE	VAN.	APR.	1	15.6	9.4	6.2	0.44	0.19	0.19
JUNE	VAN.	MAY	1	15.6	12.2	3.4	0.47	0.14	-0.14
JUNE	VAN.	JULY	1	15.6	17.2	1.6	0.44	0.15	-0.14
JUNE	VAN.	AUG.	1	15.6	17.2	1.6	0.37	0.09	0.09
JULY	OW.	JULY	1	17.2	20.7	3.5	0.40	0.20	0.20
JULY	VAN.	APR.	2	17.2	9.4	7.8	0.68	0.42	0.42
JULY	VAN.	MAY	2	17.2	12.2	5.0	0.23	0.09	0.07
JULY	VAN.	AUG.	2	17.2	17.2	0.0	0.60	0.30	0.30
JULY	SAS.	JULY	1	17.2	11.1	6.1	0.42	0.19	0.19
AUG.	OW.	JULY	2	17.2	20.7	3.5	0.67	0.21	0.21
AUG.	VAN.	APR.	1	17.2	9.4	7.8	0.37	0.13	-0.08
AUG.	VAN.	JUNE	2	17.2	15.6	1.6	0.34	0.13	-0.03
SEPT.	OW.	JULY	2	14.4	20.7	6.3	0.47	0.12	-0.12
SEPT.	VAN.	FEB.	2	14.4	5.0	9.4	0.24	0.12	0.12
SEPT.	VAN.	MAR.	2	14.4	6.1	8.3	0.51	0.26	0.26
SEPT.	VAN.	OCT.	2	14.4	10.6	3.8	0.54	0.24	0.24
SEPT.	SAS.	JULY	2	14.4	19.4	5.0	0.55	0.21	-0.21
OCT.	OW.	JULY	2	10.6	20.7	10.1	0.63	0.16	-0.16
OCT.	VAN.	FEB.	2	10.6	5.0	5.6	0.48	0.22	-0.21
OCT.	VAN.	MAR.	2	10.6	6.1	4.5	0.25	0.09	-0.05
OCT.	VAN.	NOV.	2	10.6	6.1	4.5	0.13	0.06	0.06
OCT.	SAS.	AUG.	2	10.6	17.8	7.2	0.26	0.09	-0.06
NOV.	OW.	JUNE	2	6.1	18.2	12.1	0.37	0.05	0.01
NOV.	OW.	JULY	1	6.1	20.7	14.6	0.64	0.10	0.09
NOV.	OW.	OCT.	2	6.1	8.7	2.6	0.31	0.05	0.05
NOV.	VAN.	FEB.	3	6.1	5.0	1.1	0.69	0.32	-0.06
NOV.	VAN.	MAR.	3	6.1	6.1	0.0	0.40	0.16	-0.14
NOV.	VAN.	APR.	1	6.1	9.4	3.3	0.55	0.20	-0.20
NOV.	VAN.	JUNE	1	6.1	15.6	9.5	0.32	0.10	-0.05
NOV.	VAN.	AUG.	1	6.1	17.2	11.1	0.52	0.16	-0.16
NOV.	VAN.	OCT.	1	6.1	10.6	4.5	0.36	0.13	-0.11
NOV.	SAS.	AUG.	2	6.1	17.8	11.7	0.53	0.20	-0.20

NOTE: The AAB constants for Vancouver - December did not represent (or model) any of the other 35 months of weather data.

TABLE F3

RELATIONSHIP BETWEEN CLOSENESS OF TWO MONTHS MEAN MONTHLY TEMPERATURES AND WHETHER OR NOT THE DEFINING CONSTANTS FOR ONE REPRESENT THE WEATHER DATA OF THE OTHER.

CITY: SASKATOON

MONTH FOR WHICH A&B CONSTANTS ARE USED TO MODEL SIMILAR MONTHS (MONTH A)	CITY MODELLED (MONTH B)	MONTH MODELLED (MONTH B)	NUMBER OF SIGNIFICANT FIGURES USED TO DEFINE A&B CONSTANTS FOR (MONTH A)	MEAN MONTHLY TEMPERATURE FOR (MONTH A) (°C)	MEAN MONTHLY TEMPERATURE FOR (MONTH B) (°C)	MEAN MONTHLY TEMPERATURE DIFFERENCE (°C)	ERRORS BETWEEN CURVE DEFINED BY A&B CONSTANTS FOR (MONTH A) AND DATA FOR MONTH B		
							MAXIMUM INTERPOLATION ERROR (°C)	AVERAGE ERROR (°C)	AVERAGE TOTAL ERROR (°C)
JAN.	SAS.	NOV.	3	-17.8	-6.1	11.7	0.60	0.34	-0.05
JAN.	SAS.	DEC.	1	-17.8	-13.3	4.5	0.67	0.33	0.25
FEB.	OW.	JAN.	3	-14.4	-10.9	3.5	0.56	0.25	0.05
FEB.	SAS.	JAN.	2	-14.4	-17.8	3.4	0.67	0.29	-0.19
FEB.	SAS.	DEC.	1	-14.4	-13.3	1.1	0.67	0.33	0.25
MAR.	OW.	AUG.	1	-7.2	19.3	26.5	0.67	0.13	-0.12
MAR.	SAS.	MAY	1	-7.2	11.7	18.9	0.68	0.18	-0.04
APR.	OW.	APR.	4	3.9	5.6	1.7	0.70	0.29	-0.13
APR.	OW.	MAY	3	3.9	12.4	8.5	0.38	0.17	0.03
APR.	OW.	JUNE	2	3.9	18.2	14.3	0.58	0.12	-0.05
APR.	OW.	AUG.	2	3.9	19.3	15.4	0.28	0.09	0.04
APR.	SAS.	MAY	2	3.9	11.7	7.8	0.47	0.10	0.08
APR.	SAS.	JUNE	3	3.9	16.1	12.2	0.46	0.21	0.20
APR.	SAS.	SEPT.	3	3.9	11.1	7.2	0.37	0.15	-0.03
MAY	OW.	APR.	3	11.7	5.6	6.1	0.58	0.29	0.04
MAY	OW.	MAY	3	11.7	12.4	0.7	0.55	0.25	0.23
MAY	OW.	JUNE	1	11.7	18.2	6.5	0.27	0.13	0.11
MAY	OW.	JULY	1	11.7	20.7	9.0	0.48	0.09	-0.05
MAY	OW.	AUG.	1	11.7	19.3	7.6	0.37	0.18	0.18
MAY	OW.	SEPT.	1	11.7	14.6	2.9	0.55	0.17	0.02
MAY	SAS.	APR.	3	11.7	3.9	7.8	0.59	0.21	0.13
MAY	SAS.	JUNE	2	11.7	16.1	4.4	0.59	0.27	0.24
MAY	SAS.	SEPT.	3	11.7	11.1	0.6	0.45	0.19	0.17
JUNE	OW.	MAY	2	16.1	12.4	3.7	0.57	0.18	-0.15
JUNE	OW.	JUNE	1	16.1	18.2	2.1	0.07	0.02	0.02
JUNE	OW.	AUG.	1	16.1	19.3	3.2	0.16	0.07	0.07
JUNE	VAN.	JAN.	3	16.1	2.8	13.3	0.69	0.39	0.39
JUNE	SAS.	APR.	2	16.1	3.9	12.2	0.68	0.18	-0.15
JUNE	SAS.	AUG.	1	16.1	17.8	1.7	0.61	0.20	-0.20
JUNE	SAS.	SEPT.	2	16.1	11.1	5.0	0.60	0.19	-0.18
JULY	OW.	JULY	1	19.4	20.7	1.3	0.61	0.26	0.26
JULY	OW.	OCT.	2	19.4	8.7	10.7	0.67	0.27	0.27
JULY	SAS.	AUG.	2	19.4	17.8	1.6	0.44	0.26	0.26
AUG.	OW.	JUNE	2	17.8	18.2	0.4	0.30	0.16	0.16
AUG.	OW.	JULY	2	17.8	20.7	2.9	0.45	0.09	-0.06
AUG.	OW.	AUG.	2	17.8	19.3	1.5	0.42	0.22	0.22
AUG.	OW.	SEPT.	2	17.8	14.6	3.2	0.29	0.17	0.17
AUG.	SAS.	DEC.	1	17.8	-13.3	31.1	0.67	0.33	0.25
SEPT.	OW.	MAY	2	11.1	12.4	1.3	0.38	0.20	0.09
SEPT.	OW.	AUG.	2	11.1	19.3	8.2	0.39	0.11	0.03
SEPT.	SAS.	APR.	2	11.1	3.9	7.2	0.47	0.21	-0.01
SEPT.	SAS.	JUNE	2	11.1	16.1	5.0	0.58	0.28	0.28
SEPT.	SAS.	DEC.	1	11.1	-13.3	24.4	0.67	0.33	0.25
NOV.	SAS.	JAN.	4	-6.1	-17.8	11.7	0.64	0.27	0.02
DEC.	OW.	JULY	1	-13.3	20.7	34.0	0.17	0.08	0.07
DEC.	OW.	AUG.	3	-13.3	19.3	32.6	0.69	0.15	-0.01
DEC.	VAN.	MAR.	1	-13.3	6.1	19.4	0.66	0.40	-0.04
DEC.	VAN.	OCT.	1	-13.3	10.6	23.9	0.67	0.36	0.07
DEC.	VAN.	NOV.	1	-13.3	6.1	19.4	0.67	0.33	0.07
DEC.	SAS.	JAN.	2	-13.3	-17.8	4.5	0.54	0.20	0.07
DEC.	SAS.	JULY	1	-13.3	19.4	32.7	0.25	0.07	0.00
DEC.	SAS.	AUG.	1	-13.3	17.8	31.1	0.37	0.22	0.22

NOTE: The A&B constants for Saskatoon - October did not represent (or model) any of the other 35 months of weather data.

APPENDIX G

MEAN MONTHLY TEMPERATURES FOR
OTTAWA, VANCOUVER AND SASKATOON

TABLE G1												
MEAN MONTHLY TEMPERATURES (°C) = Tmm												
CITY/MONTH	J	F	M	A	M	J	J	A	S	O	N	D
OTTAWA	-10.9	- 9.5	-3.1	5.6	12.4	18.2	20.7	19.3	14.6	8.7	1.4	- 7.7
VANCOUVER	2.8	5.0	6.1	9.4	12.2	15.6	17.2	17.2	14.4	10.6	6.1	4.4
SASKATOON	-17.8	-14.4	-7.2	3.9	11.7	16.1	19.4	17.8	11.1	5.0	-6.1	-13.3

POOR

QUALITY

ORIGINAL